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THEORETICAL ELEMENTS
OF
ELECTRO-DYNAMIC MACHINERY.

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P R E F A C E .

THIS volume is a collection of a series of articles recently published in the "Electrical Engineer" of New York.

The writer's desire and intention has been to develop for students of electrical engineering the applied or arithmetical theory of electro-magnetism, as distinguished from the purely mathematical theory of this great and important subject. The groundwork only can be said to have been completed within the limits of these covers.

THEORETICAL ELEMENTS OF ELECTRO-DYNAMIC MACHINERY.

CHAPTER I.

Magnetic Flux.

MAGNETISM is the science that deals with a series of phenomena whose ultimate nature is unknown, but which result from, or are at least accompanied by, a particular kind of stress. This stress may reside in matter or in the air-pump vacuum. The magnetic metals, iron, nickel, and cobalt, when submitted to this stress, not only intensify it in their own substances, but are strained in such a manner as to sustain the stress independently to a greater or less degree after the existing cause has been withdrawn. In other words they become magnetic and remain magnetized.

The only known sources of magnetic action are three, viz., electric currents, electrical charges in translatory motion, and magnets.

Any magnetized space, or region pervaded by magnetic stress is a "magnetic field," but the term is commonly applied to the space which separates the poles of an electromagnet. The existence of stress in the field is evidenced in several ways :

1. By the magnetic attraction or repulsion of magnetized substances introduced within the field.

2. By the influence exerted on the molecular structure or molecular motion of a number of transparent substances whereby the plane of propagation of luminous waves is rotated in traversing them.

3. By the electromagnetic energy which is found to be absorbed by the medium during the establishment of a magnetic field, stored there while the field is maintained, and released at its subsidence.

4. By the electromotive forces that are found to be generated in matter moved through the field.

These experimental evidences point to the action of magnetic stress pervading the magnetized medium. Moreover the stress never terminates at an intersecting boundary, but follows closed paths. A line of stress is a closed loop like an endless chain. If a small compass-needle were introduced into a magnetic field, and kept advancing from point to point in the direction it assumed at each instant, it would finally return to the position from which it started. This can only be shown in the case of fields established by electric currents in wires, or coils of wire, as the needle could not complete its circuit through the mass of an iron magnet. The circuital distribution of the stress indicates its appurtenance to the category of fluxes. That is to say, in any magnet, or magnetized region, there is a distribution of influence analogous to the flow of current in an electric circuit, or the flow of water in a closed pipe or re-entering channel. The marked distinction, however, between the flow of magnetism, and the flow of electricity or of fluid material, lies in the fact that no work is done and no energy exchanged in the passage of the magnetic current. The electric current and the moving liquid encounter resistance and develop heat in moving against that resistance, but the magnetic flux acts as electrical currents or material currents might act, if unchecked by resistance but regulated in quantity by other limitations; and if water were itself frictionless, and were set circulating in a closed frictionless pipe, it would necessarily continue in perpetual motion. It of course by no means follows that any circulating motion of matter, or of ether, actually takes place in a magnetic circuit. That there is a possibility of such motion is a consideration left to the theory of the ultimate and fundamental nature and origin of magnetism. All that is essential to the conception of a magnetic flux is a continuous stress acting along a closed path or circuit, such that a single magnetic pole (if such could exist alone, or let us say as the nearest representation in fact, one pole of a long bar magnet) were introduced into this path it would be continually urged round the circuit.

Flux in a magnetic circuit, just as in a hydrostatic, or electric circuit, possesses at each point intensity as well as direction, and can therefore be completely specified by a vector. According to the conventions established in the absolute c. g. s. system, a field of unit intensity will exert unit pull or one dyne, upon an isolated unit magnetic pole introduced therein. The intensity of the earth's magnetic flux is approximately 0.6 unit in the open country around

New York, and consequently, a single north-seeking or unit pole suspended in that neighborhood would be drawn downwards in the direction of the dipping needle with a force of 0.6 dyne, which would represent the weight of nearly 0.6 milligramme (a dyne being about 2 per cent. greater than the earth's gravitation pull on a milligramme of matter). Practically it is impossible to obtain an isolated unit pole, but it is quite possible to measure the pull exerted by a magnetic field upon some definite system of electric currents or magnets, and to deduce what the corresponding pull would be on a unit magnetic pole. This would be the numerical intensity of the flux in the neighborhood of the point considered. Flux intensity is denoted by the symbol B , and is, strictly speaking, a vector denoting direction as well as magnitude.

The direction of a magnetic flux is the direction in which it would move or tend to move a free north-seeking pole. The north or blue end of a small compass needle points in the direction of the field surrounding it. According to this convention it follows that the earth's flux is from the geographical south towards the geographical north pole. Also,

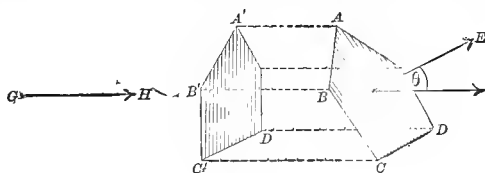


FIG. 1.

to follow the direction of a flux, is to move along it in a positive direction, while to oppose it is to move negatively.

The total quantity of flux that will pass through a given normal plane area, is the product of that area and the intensity when the latter is uniform. For example, a plane area of say 150 sq. cms. held near New York perpendicularly to the direction of the dipping needle will contain 150×0.6 , or 90 units of flux. In ordinary language, it will contain 90 lines of force or of induction. The word induction, however, from frequent misapplication is apt to be so ambiguous that it is advantageous to dispense with it in this sense.

If the intensity varies from point to point, the total flux will be the average intensity over the surface, multiplied into the area. If the plane area does not intersect the flux at right angles the flux enclosed will be the average intensity, multiplied by the area of the boundary as projected on a plane

intersecting perpendicularly, which might be termed the equivalent normal area. This is shown in Fig. 1 where $ABCD$ represents an arbitrary boundary drawn on a plane surface whose normal $\mathbf{n}$ makes an angle θ with the vector $\mathbf{GH}$, and this vector represents the uniform flux intensity in direction and magnitude. The total flux enclosed by $ABCD$ will be the product of the length $\mathbf{GH}$ into the area $A'B'C'D'$ which is the projection of $ABCD$ on a plane normal to $\mathbf{GH}$. Calling the area $ABCD$ a , φ the total flux, and B the intensity, it is evident that

$$\varphi = a B \cos \theta$$

Finally if B , instead of being uniform throughout the space covered by the area, varies from point to point, the total flux could be found by dividing the areas into a sufficiently large number of small portions, determining the values of θ and B for each. By taking the area small enough, θ and B would be more and more nearly uniform within the limits of each, and the flux through each could then be determined separately. The sum of these fluxes would be the total flux enclosed by the whole boundary area. In other words the total flux would be the surface integral of the normal intensity all over the area as expressed by the equation

$$\varphi = - \int S. Bn \, ds.$$

This general result is independent of the form of the surface round which the boundary is described. This surface may be plane, warped, or convoluted.

As an example, consider the spherical surface represented in Fig. 2. Let this surface be definitely located in any permanent field—in a flux that does not vary with time—but which may be quite irregular in intensity. We may assume that the vector B is known or can be determined for every point. Draw any closed line $CDEG$ round the sphere. Then if this sphere does not enclose any source of magnetism—current or magnet—we may follow out the plan above indicated and take the surface integral of flux: (1) over the lesser spherical area $CDEGH$; (2) over the greater spherical area $CDEGK$; (3) over a surface stretched tightly across the boundary; or (4) over any conceivable surface into which the diaphragm $CDEG$ could be expanded. The result will in each case be the same total flux, if due precaution be taken to attach + signs to emerging + flux, and — to an entering + flux. Any element of flux not passing through the boundary must then

cut the surface an even number of times, half in entrances and half in emergences, and the summation of these ele-

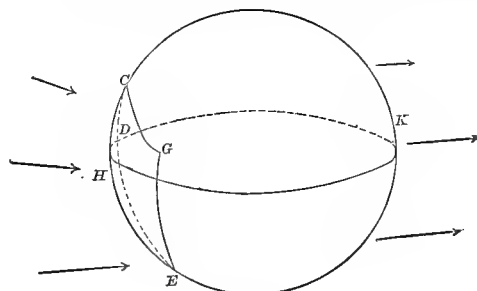


FIG. 2.

ments having opposite signs will cancel out from the result in pairs.

This proposition, capable of indirect experimental verification, is tantamount to the statement that magnetic flux neither expands nor condenses in its passage through bodies, however its direction or intensity may vary. Because the flux which is steady in regard to time must be entering into and emerging from any portion of space at the same rate, unless at that moment, expansion, condensation, generation, or annihilation be taking place within the confines. In the above instance of Fig. 2, it is evident that the flux which enters the area CDEG in the direction of the arrows must be issuing from the volume CDEGK at the same rate if accumulation or dissipation is not at work within, and the quantity which in any given time issues from the spherical surface without having also entered there, must have come in through the boundary CDEG. In this respect magnetic flux behaves like the flow of an incompressible fluid.

CHAPTER II.

Magnetomotive Force and Potentials.

ANY steady distribution of magnetic flux is subject to a potential except within the substance of conductors carrying electric currents. That is to say, it is possible to assign to each point of the field a numerical value, such that its gradient, or rate of change in any direction, shall represent the corresponding component of flux intensity in that line. In recognizing a distribution of potential in space, we therefore erect everywhere an imaginary numerical scaffolding, each point being invested with its appropriate magnitude and sign. The direction of maximum gradient in the scaffolding at any point, is the direction of the flux, the steepness of the grade being the flux intensity. The plane normal to the direction of maximum gradient will also be the local plane of no gradient or of level potential. If surfaces are formed by the colligation of all points having equal potentials, they will be equipotential surfaces. There will be no flux in the plane tangential to such a surface, and the whole flux will be normal to it. It is also evident that the steeper the grade, the closer will be the successive equipotential surfaces, just as in the case of contour lines on a map of levels; so that where the flux density is great the surfaces follow in rapid succession, spreading apart again where the intensity is low.

In fact, the intensity is the reciprocal of the distance between adjacent surfaces whose potential differs by unity, as represented by the equation :

$$B = - \frac{dP}{dn};$$

where P is the potential and n the distance measured along the normal to the equipotential surface, a condition condensed into the notation

$$B = - \nabla P.$$

The negative signs are here appended in order to preserve the convention that the direction of the flux shall be down grade, or the direction in which the potential falls.

If the maximum gradient of potential at a given point is at the rate of, say, 500 units (ergs) per centimetre of distance, the flux density at the point will be 500 c. g. s. lines per square centimetre. As the conception of magnetic potential has important practical applications, we shall proceed to examine its distribution in a few cases.

The simplest instance is probably that of an indefinitely long straight wire of circular section carrying a steady

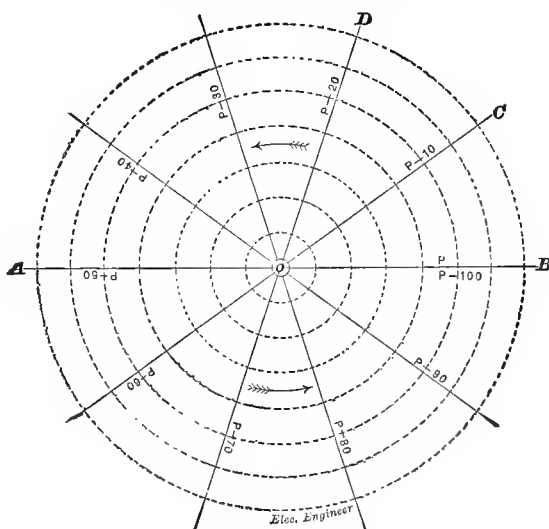


FIG. 3.

current. Fig. 3 represents a plane drawn perpendicularly through such a wire, carrying a current of $100 / 4\pi$, or 7.958 c. g. s. units (79.58 amperes, since the absolute unit of current in the electromagnetic system is 10 amperes). The reason for selecting this particular current strength is merely to secure graphical simplicity. The current is supposed to be passing upwards through the wire from the plane towards the reader. The equipotential surfaces for such a wire are radial planes through the axis. They commence, however, not at the axis itself, but at the surface of the wire, because within the substance of the conductor there is no potential, and they have no exterior boundary, or, in other words, terminate at infinity. The traces of 10 such radial planes are shown equidistant in the diagram,

and on each of these the potential has a constant value. Selecting any one plane as $o B$, we find, however, no determination of its absolute numerical value from the conditions of the problem, and we may therefore assign to this plane any numerical value we please, positive, negative, or zero; but having once selected it, the whole system immediately assumes definite numerical configuration.

Call the potential of $o B$, P , then the conditions determine that the potential of $o C$ will be $P - 10$; that of $o D$ will be $P - 20$; and so on. Between any two successive planes here represented, it would of course be possible to insert nine equidistant and intermediate planes whose potentials would differ by one unit in succession, while any number of other planes might again be crowded between the unit planes, representing definite intermediate fractional potentials.

Since the flux density is the rate of fall of potential, it is clear that B must have the direction down grade, as shown by the arrows, and that it must be at each point normal to the radial plane or tangential to the circle drawn through the point and concentric with the wire. The flux will thus be confined to concentric circles, and a few of these whose number is unlimited are represented in the figure by the dotted lines. A free north-seeking pole brought to any point of the field would be carried by the flux round the circle passing through that point in the direction of the arrows.

It is evident from the figure that the distance between any pair of equipotential planes increases uniformly with the distance from the axis; and since the flux is inversely proportional to the distance of these planes, it follows that the intensity varies inversely as the distance from the axis.

If we follow the potential down grade from plane to plane, and complete a circle round the wire, we find that the plane $o B$, which at the outset had the potential P , now assumes the potential $P - 100$. Also, a second circuitation finds this potential changed to $P - 200$, and so on, for any number of turns. The potential of any plane is thus capable of an indefinite number of values successively differing by 100. In other words, the potential is cyclic and multiplex. The gradient is definite, but the absolute elevation above datum is indeterminate. This cyclic property is characteristic of magnetic potentials, which are typically radial-planed, as opposed to electric or gravitational potentials, which are typically represented by successive envelopes round a common nucleus.

MAGNETOMOTIVE FORCE.

The simple law of potential, applicable to the case of a long current-conveying wire, is that the magnetomotive force in any loop surrounding a steady current of c c. g. s. units is $4 \pi c$. This important quantity, magnetomotive force is that which produces flux. It is convenient to represent it for brevity by the initials *M. M. F.* in imitation of the analogous term electromotive force in the voltaic circuit, and its recognized symbol *E. M. F.*

M. M. F. and flux standing to each other in the relation of cause and effect, one cannot exist without the other. In the great majority of cases with which we are familiar, the flux is subject to a potential such as we have above traced, and in all these cases the *M. M. F.* between any two points is essentially and numerically, the difference of potential existing between them. The *M. M. F.* may be stated as producing the difference of potential, or to be the equivalent of the difference of potential itself; but just as in the voltaic circuit, it is possible to have flux where no system of potentials can exist, and it is advisable to observe that potential differences are only a sub-class of motive force. For example, we have observed that no system of magnetic potentials exists within the substance of the long, straight, current-conveying wire; yet there is flux within it at every point, except on the axis itself, and consequently there must be *M. M. F.*, but no such distribution that its space gradient corresponds to the flux.

Outside the wire, the *M. M. F.* or total *P. D.* (potential difference) is $4 \pi c$ for one complete circuitation, and since the potential at any point is dependent only on the angle contained between the radial planes through that point and some point of reference, it is clear that the potential varies by $\pm \pi c$ for each revolution of 2π , or by $2 c$ per radian in circular measure, $\left(\frac{4 \pi c}{360} \right.$ per degree of common measure.)

Let the plane through *o b* be the plane of reference, θ and r the polar co-ordinates of any point a . Then having P the potential assumed for *o b*, and P_a the potential at *o a*, we have :

$$P_a = P - 2 c \theta.$$

Again, to find the flux intensity at a we have only to differentiate the potential there in the direction along the path of flow, or tangential to the circle, thus :

$$B_a = - \frac{d P_a}{d s} = - \frac{d}{d s} (P - 2 c \theta) = - \frac{d}{d \theta} (P - 2 c \theta) \cdot \frac{d \theta}{d s}.$$

Also, $\theta = \frac{s}{r} \therefore \frac{d\theta}{ds} = \frac{1}{r}$, while P is constant and

$$B_a = \frac{2c}{r}.$$

From this it follows that the c. g. s. flux at any point near a long straight wire carrying a current is equal to the c. g. s. current strength divided by half the distance from the axis of the wire in centimetres, and is directed tangentially to a circle drawn through the point, concentrically with the wire, in the normal plane.

As a practical application of this result, we can readily determine the deviating influence of such a current-conveying conductor, on a small magnet suspended in its vicinity. Suppose the needle suspended horizontally about its axis directly over and 40 centimetres away from a very long horizontal wire lying in the magnetic meridian, and carrying a current of 20 amperes (2 c. g. s. units), the earth's horizontal component of flux being 0.2 c. g. s.

The flux intensity due to the current at the position occupied by the needle will be $\frac{2c}{r} = \frac{2 \times 20}{40} = 0.1$. Each pole

of the needle will therefore be acted upon by two horizontal component fluxes, one along the meridian plane due to the earth's M. M. F., and = 0.2, the others at right angles to the meridian plane, due to the M. M. F. of current, and = 0.1.

The resultant of these rectangular components will be $\sqrt{0.04 + 0.01} = \sqrt{0.05} = 0.2236$, and its direction will make an angle $\tan^{-1} \frac{0.1}{0.2} = \tan^{-1} 0.5 = 26^\circ.34'$ with the

magnetic meridian. The needle will assume the local direction of resultant flux, and deflect through this angle on the establishment of the current. The direction of deviation will, of course, depend on the direction of the current in the wire, but the deviation of the north pole will always be related to the direction of the current, in the same manner that the rotation of a nut is related to its retreat or advance upon an ordinary right-handed screw.

The case in which the magnetic equipotential surfaces are parallel, equidistant planes is of considerable importance, on account of its geometrical simplicity, and also of its practical applications. The gradient being everywhere equal and in the same direction, B is everywhere equal in intensity and normal to these planes within the limits of the distribution. This condition is termed a uniform field, and must necessarily be confined to limited portions of any

magnetic circuit, when it is considered that all flux lines are closed curves, and cannot follow straight lines for even one-half of their total circuital length. On the other hand, even the most irregular and sharply curved fields can be divided into so great a number of small elements that each may be considered separately as a uniform field. The earth's magnetic field represented over the surface of a small model globe would appear to be far from uniform, but on the actual scale uniformity appears to exist, and B is constant (at any one moment, beyond the range of iron or iron ore) within several hundred feet of any locality so far as the most sensitive instruments can discern.

A uniform helix of wire was named by Ampere a solenoid. Strictly speaking the term only applies to a series of equal and parallel conducting rings arranged in close successive order round a common axis, but the difference between a closely wound spiral and the hypothetical arrangement is usually negligible in action. The field within an infinitely long solenoid carrying a current is uniform, the equipotential surfaces being planes normal to the axis, and dividing the interior of the solenoid into equal compartments.

If c be the c. g. s. current circulating in the solenoid, and n the number of turns per centimetre of length, nc will be the current turns per unit of length, and the distance between the successive integral equipotential planes will be $1/(4\pi nc)$ cms. The gradient being one erg in this distance, its representative, the flux, will be $1 \div 1/(4\pi nc)$ or $4\pi nc$ units of flux per square centimetre, and if the cross-sectional area of the solenoid be a square centimetres, the total flux within it will be $4\pi nca$.

While theoretical uniformity of field would only be attainable within an infinite solenoid, a very close approximation to uniformity is secured from solenoids of convenient practical dimensions if their length is great compared with their diameter. The flux becomes divergent in such cases towards the extremities, and as the length of the helix is reduced, the limits of practical uniformity are retracted and narrowed to the centre of the coil. The flux emerging from the extremities completes the external circuit by indefinitely extending contours.

Fig. 4 represents a plane section through the axis of a solenoid $A O B$, and the ovals are the traces of a few of the equipotential surfaces that govern the external flux. If these could be rotated about the axis $A O B$, a dumb-bell-shaped system of equipotential envelopes would be formed. The length $A B$ being taken as 100 cms., and the c. g. s.

current turns 100 per centimetre, the successive integral surfaces would be represented to scale from 5 ergs positive to 5 ergs negative, the central plane through O , normal to the axis, being that of zero potential. The sectional area of the solenoid is also assumed to be one square centimetre for convenience. The flux is always at right angles to these surfaces on its passage down grade from B to A , as indicated by a single dotted line $A C B$, taken as one path at random out of the infinite number possible. Within the

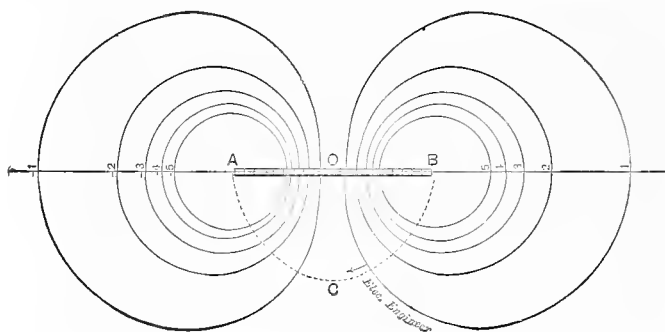


FIG. 4.

solenoid, the surfaces are very nearly planes, and the field very nearly uniform from the centre O to within ten diameters of the ends.

Another form of conducting surface which produces a uniform field within the entire space it envelops is Maxwell's ellipsoid. If an ellipsoid be divided into slices by equidistant planes normal to one of the axes, and the surface between each consecutive pair be uniformly wound with the same number of turns of conductor, so that the current turns round each slice are equal, the field within the ellipsoid will be uniform. The proposition applies to a sphere as a particular case. Fig. 5 represents a plane section through the centre of such a sphere and gives the traces of a few equipotential curves. The diagram if revolved about the axis $A O B$ would generate a system of equipotential surfaces forming planes within the hollow sphere, and spheroidal envelopes external to it. If the diameter of the sphere be 20 cms. and the surface be wrapped with ten turns of wire per centimetre of axis, making 200 turns in all, a current of 0.002357 c. g. s. will establish to scale the surfaces represented; and generally,

if nc be the current turns per axial centimetre, the potential of any plane through the sphere parallel to the windings, distant x centimetres from the centre, will be $\frac{8\pi}{3} ncx$, a value independent of the diameter of the sphere.

The gradient will then be $\frac{8\pi}{3} nc$ ergs per cm. and this will be the value of B everywhere within the sphere.

This case of the uniformly wound ellipsoid seems to be the only one yet known that will produce throughout the

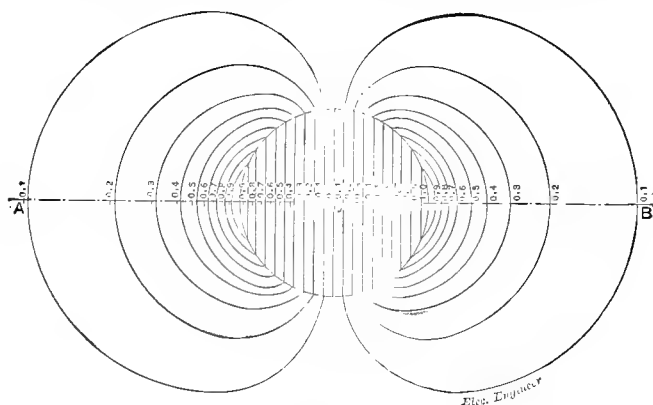


FIG. 5.

interior of a conducting surface having any assigned finite magnitude a perfectly uniform field. The geometrical difficulties in the way of its accurate practical construction appear to have excluded its application from the purposes of measurement.

It is curious to observe that while in Fig. 4 there is an absolute discontinuity at the surface of the solenoid between the internal and external potential surfaces except at the centre o where the zero planes coincide without and within, in Fig. 5, at the surface of the sphere, there is a discontinuity of direction only, and a continuity of numerical value. The surface continues with a sudden change in shape.

We have defined the potential as that quantitative property of a point in space, whose gradient is the vector flux intensity. There is, however, another and useful point of view from which the conception of potential can be regarded, and which has in reality prior claims to considera-

tion so far as regards the historical development of the subject. The definition which follows can, however, be readily proved to be involved by, and equivalent to, the preceding.

The magnetic potential at any point, due to any magnetic body or system, is the work which would have to be mechanically performed solely against magnetic forces, in conveying a free unit north pole from an infinite distance up to that point, assuming that no other magnetic forces are active on the pole during the journey except those exerted by the body or system considered.

In other words, the potential of a magnetic element or system at a point is the potential energy between that system or element and a free unit north or positive pole there deposited; for, if left free to move in obedience to magnetic forces, the pole would commence to retreat and develop kinetic energy that would find its complete value in a culminating velocity only after having receded to an infinite distance. That final kinetic energy would be the potential of the starting point. That is on the supposition that the potential of the point was positive. If, on the contrary, the potential had been negative, the $+$ pole would not have retreated, but would have absorbed work, *i. e.*, developed negative energy during the passage; it would have been attracted by the body exercising potential influence and not repelled.

As a consequence of this relation, it is evident that if we move our ideal free north unit pole from a point whose potential is, say, 50 ergs $+$, to one whose potential is $+75$ ergs, we shall have to expend 25 ergs of energy in the process. The energy will be stored in the system potentially, and will be liberated on a return journey. The amount of energy is independent of the path selected between the points, provided that it is not linked with any current or magnet, or in other words that the path could be drawn tight into a straight line without cutting through any current or magnet. For if a rectilinear path expended 25 ergs, while another course diverged to execute one complete turn, in addition, round some current of *c c. g. s.* units, the work done in this passage would be $25 \pm 4 \pi c$ ergs, the sign depending on the direction of circuitation and of this current.

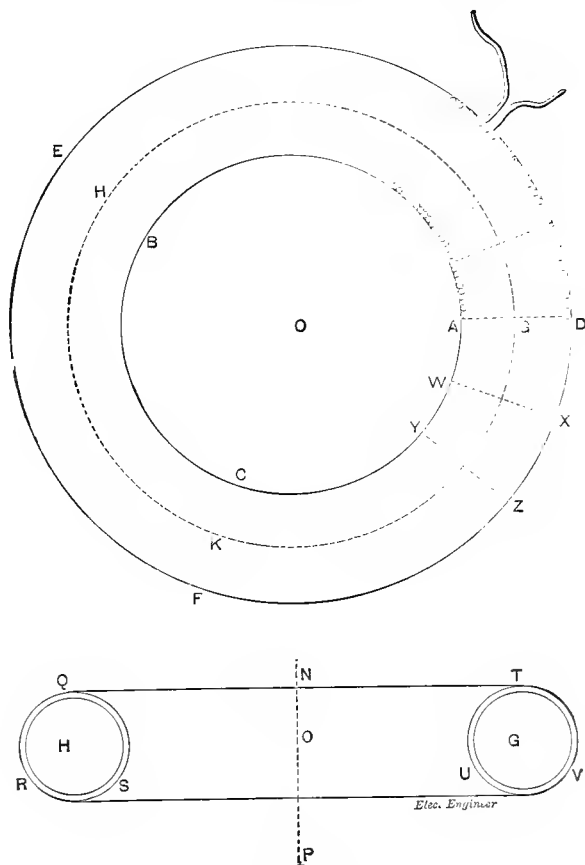
Also it is evident that no work would be expended magnetically on, or by, a pole constrained to move exactly over an equipotential surface. Whatever energy might be exchanged between the pole and its environment under those

conditions would be altogether independent of the magnetic system, and would take place equally if that system had been suppressed. On this account a pole does no magnetic work in directly approaching or leaving an infinite rectilinear current alone in space, and a long, straight current-conveying wire does not attract a pole, for we have seen that the motion would be confined to some equipotential plane. But work would commence to be done, and energy commence to enter or leave the system so soon as the pole departed from the straight line of approach, and moved round the wire.

An important system of flux and potential distribution is that established within an endless solenoid, or what is practically a helix with its two extremities united. Such a solenoid completes its magnetic circuit entirely within itself and has no exterior flux. It therefore has no external magnetic potential. In the practical application of this case, attention has sometimes to be paid to the departure from the strict definition of a solenoid, for a helix is virtually a solenoid plus a conductor along the axis, just as motion along a spiral is circular plus a translatory advance in the axial line; so that a closed current-conveying helix, however closely wound, and nearly uniform, will exercise some attractive influence upon a magnetic needle, and thus evidence an external flux, unless before the extremities of the coil are joined, one wire be led back to its starting point parallel to the axis, a proceeding which will cancel out the axial component except at very close range. At all ordinary distances, the closed helix will then be deprived of external magnetic influences, no matter how tortuous the curve its axis may follow.

Within this closed solenoid there will be a distribution of magnetic potential, but it will be complex, as also the flux it controls, and difficult to represent graphically or numerically, unless the axis of the solenoid lies upon a circle. In this case the distribution is simple. The solenoid will then assume the form of an anchor ring, represented in plan and diametral sections by Figs. 6 and 7, where $N O P$ is the axis of revolution, $Q R S$, and $T U V$, sections of the helix surrounding the coil axis $G H K$. In such a system the equipotential surfaces are planes drawn radially through the axis of revolution, but confined entirely to the interior of the ring, such as $AS AD$, WX , and YZ . The $M. M. F.$ active within the ring is 4π times the current turns linked with it. If there were 180 turns of wire on the ring, as shown, and a current of 2 amperes circulating, or 0.2 C. G. S.

units, the M. M. F. would be $4\pi \times 180 \times 0.2$, or 45.25 ergs per unit pole.¹ This will be the total difference of potential for one complete circuitation within the ring, representing a rate of change of potential of 0.1257 erg on unit



FIGS. 6 AND 7.

pole per degree of revolution round the axis NOP . This gradient is definite, but the potential at any plane is

1. The dimensions of magnetic potential being $M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$, magnetic pole $M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$ and energy $M L^2 T^{-2}$, it is important to observe that potential is equivalent to energy $\div$ pole, and when expressed in ergs, implies ergs expended on or by a unit pole.

arbitrary, and may be selected at will as a starting point. Once chosen, the potential is evidently cyclic, and increases or diminishes by $4 \pi n c$ ergs in each revolution, according to the direction of circuitation. If there is more than one layer of wire on the ring, and each layer be uniformly wound, the system may be considered as a combination of independent solenoids, one for each layer. The potential in the interior will be that due to the sum of the solenoids, while at any point in the winding space the potential will be that due to the layers outside, those within the point's axial radius having no influence beyond their surface.

Within the anchor ring, the flux, necessarily perpendicular to all the successive equipotential planes must follow circles concentric with the axis of revolution NOF . The intensity will be uniform along any one circle, and will be equal for equal circles, but it would be different in circles of different radii, being weakest at the outer side DEF , and strongest at ABC , varying inversely as the radius of revolution. With reference to what has appeared on page 7, concerning the field surrounding an infinite straight conductor, it is evident that any steady current c making n uniform turns round an anchor ring of wood or other non-magnetic material, produces throughout its interior exactly the same distribution of potential and flux that a current of nc units flowing along the ring's axis of revolution indefinitely prolonged would produce throughout all space. The circular solenoid produces within the finitelimits of its interior the condition that an infinite conductor would establish in infinite space.

This analogy suggests what is in fact, readily proved, that the radial planed system of potentials and circular distribution flux is not dependent upon the cross-section of the solenoid. We have thus far supposed it to be circular, but it might be square, elliptical, or of any arbitrary shape, provided that its surface was a surface of revolution, having the equivalent straight wire for its axis. The field due to the equivalent current along the infinite axis would still reside within the coil. From the results obtained on page 9, the magnitude of B would be there represented by

$$\frac{4 \pi n c}{2 \pi r}, \text{ or } \frac{2 n c}{r}, \text{ where } r \text{ is the radial distance of the point}$$

considered from the axis of revolution. In a solenoid whose cross-section is of small dimensions compared with the radius of revolution, B is nearly uniform in intensity, and if the cross-sectional area be denoted by a , the flux

within the solenoid will then be $\phi = Ba = \frac{2\pi ca}{r}$. If, however, the cross-section of the solenoid be of such shape and dimensions that the intensity varies considerably in different parts of it, the area must be multiplied by the average B to give the total flux, or by the flux intensity existing at the centre of gravity of the area, considered as a thin material lamina.

The general case of potential distribution round a current has been only determined in a comparatively small number of instances of which the foregoing are the simplest examples. Yet in order to determine the distribution of flux existing through space in the vicinity of a given system of conductors, it is usually simpler to calculate the distribution of potentials where it exists, and deduce the flux from the gradient, than to compute the general value of B directly. The difficulty in either case is essentially one of summation, for each small element of a current-conveying conductor contributes its own share to the potential or flux at a point independently of all the others, and the geometric sum of all these elementary potentials or intensities is the resultant at that point.

The rule for determining the magnetic potential of any circuit at any point is as simple as it is often difficult to numerically apply. Let a sphere of one centimetre radius be described round that point as a centre, and let a straight line commencing at this point centre move round the circuit considered, in the direction of the current. The trace of its path cut on the unit sphere will form some closed curve and the area of the spherical surface enclosed will be the solid angle subtended at the point by the circuit. This, multiplied by the strength of the current, will be the point's potential. This shows that for electric circuits comprised in one plane, the potential of that plane outside the limits of the circuit, is everywhere zero, or some multiple of 4π , for the solid angle subtended by the circuit will vanish at any point upon it. Also, if two or more loops in a circuit stand behind one another when viewed from a point, so that the radius vector makes re-entrant loops on the unit sphere in contouring them, the overlapping areas must be added to the total surface within the boundary, or, in other words, the potentials of superposed loops are superposed.

FIG. 8 represents a plane section through the axis $c o d$ of a circular loop of wire AB carrying a current¹ and the

1. J. Clerk Maxwell, *Electricity and Magnetism*, Vol. II, Plate xviii.

traces of 24 equipotential lines. If the diagram be rotated about the axis COD , these lines will generate the corresponding system of lenticular equipotential surfaces. These commence at the surface of the wire, and are all finite except the plane AOB which may be extended indefinitely.

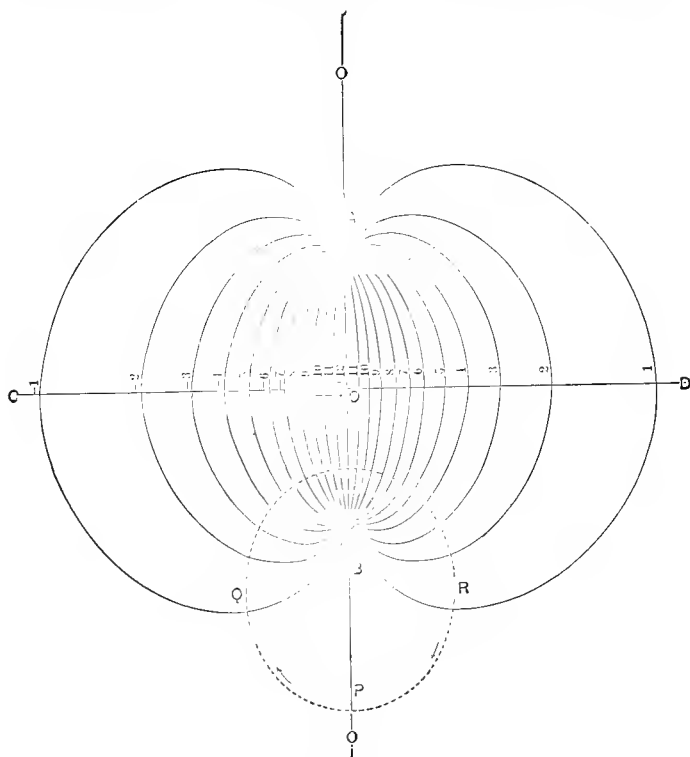


FIG. 8.

The case applies practically to a single-ring tangent galvanometer, and in order to make the particular equipotential surfaces here indicated correspond to successive integers, the current in the ring must be 1.91 c. g. s. units or 19.1 amperes, circulating clockwise as viewed from C . The M. M. F. or total P. D. in the magnetic circuit will be $4\pi \times 1.91$, or 24 ergs per unit pole, and the surfaces are marked from + 12 to - 12, down grade in the direction of the arrow.

We have already seen that each equipotential surface must be so described, that from any point in it the solid angle subtended by the ring is the same. While this rule accurately defines the potential at any distant point from which the ring appears as a mere circular filament, it needs modification for those points nearer the ring where the solid angle subtended by its outer edge and inner edge differ sensibly. At these shorter ranges, it is necessary to suppose the ring divided up into a suitably large number of smaller and filamentary circles, each carrying its due share of the current. The sum of the solid angles multiplied by their currents for all these elementary rings will give the resultant potential. The same reasoning applies to a coil of any shape made up with separate turns of insulated or uninsulated wire.

Starting from any point P on the plane $A O B$ outside the ring, the surface of the ring vanishes to the eye of an observer at this point, so that the solid angle and potential are here both zero. As we proceed down grade along the dotted line of B cutting the successive equipotential surfaces at right angles, the ring opens into view, and the solid angle,—its area of projection on a unit sphere—increases, but the direction of the current appearing clockwise, the potential is negative. It continues to increase numerically until the interior plane $A O B$ of the ring is reached, when its projection will embrace just half the surface of a unit sphere, making the solid angle 2π , and the potential $-2\pi c = -2\pi \times 1.91 = -12$. The instant, however, that the central plane is passed, the aspect of the ring changes, and the current appears to circulate counter-clockwise. The potential is now positive and $+12$ (the solid angle being still 2π). At the passage of the median plane, there has been an abrupt cyclic elevation of potential of $4\pi c$, without any discontinuity in the gradient on either side of the ascent.

This distribution of potential is depicted diagrammatically in Fig. 9. $A n c$ is the perspective view of a horizontal loop, an elementary magnetic circuit or individual line of flux such as $P Q R$ in Fig. 8. The vertical distance of the line $D C E$ above or below, marks the potential at each point of the circuit. The gradient of the line $D C E$ has everywhere a definite value, and numerically equals the intensity B . At the point o corresponding to the interior plane $A O B$ of Fig. 8, the potential rises abruptly from E to D through $4\pi c$ units, the gradient just before E , and just after D being the same.

This is precisely analogous to the conditions that would be established if $A B C$ were a galvanic instead of a magnetic circuit, with a battery of $4 \pi c$ volts at O , except that galvanic circuits are usually closed through insulated wires of uniform cross-section and conductivity, that im-

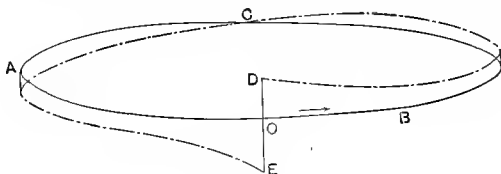


FIG. 9.

pose a uniform gradient or drop in electric potential. If the circuit were metallic—not completed through the ground—the potential would start at the copper pole of the battery with a value of $+ 2 \pi c$ volts, and would fall in one circuitation to $- 2 \pi c$ volts at the zinc pole, assuming that the internal resistance and consequent drop of potential in the battery were negligible. There would be a cyclic elevation of potential in the battery of $4 \pi c$ volts.

Fig. 10 represents a similar elementary path of flux $A B C$

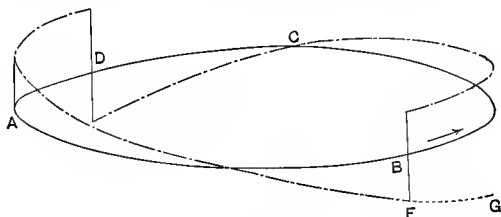
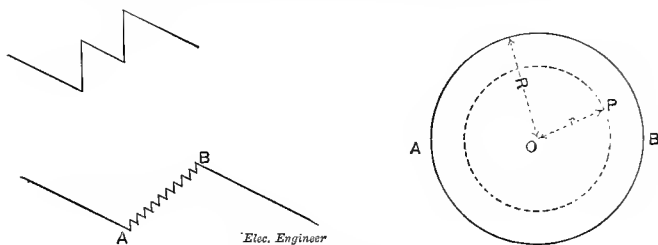


FIG. 10.

threading two rings of wire, each carrying a current of c units. The M. M. F. in the circuit will here be $2 \times 4 \pi c$ or $8 \pi c$ ergs per unit pole. There are two discontinuities of potential, at B , and at D ; while the average gradient is twice as steep as in the preceding case. It will be noticed that all discontinuity might be avoided if the potential line at F continued along the dotted line towards G instead of rising to the new level, but by pursuing this course, it would continue to generate a spiral whose successive con-

volutions would be $8 \pi c$ units apart. The potential would then be continuous but multiplex. Similar conditions would apply to the galvanic circuit, but for the fact that all electric potentials are conventionally measured relatively to that of the ground in the vicinity, which precludes multiplex values and makes cyclic discontinuity the necessary alternative.

In passing along a magnetic circuit through the spires of a coil or solenoid conveying a current c , there is a cyclic elevation of potential equal to $4 \pi c$ at the plane of each spire. For two spires the elevation would evidently be of the type shown in Fig. 11; for a larger number of closely



FIGS. 11, 12 AND 13.

associated turns, the line of elevation, instead of being abrupt, would appear to have a definite gradient, AB , Fig. 12, namely, $4 \pi n c$, n being the number of turns per centimetre of circuit. This explains why the flux, in Figs. 4 and 5 appears to move up grade, contrary to rule, within the solenoid and sphere. The apparent anomaly is due to the continuous superposition of small cyclic elevations, and the potential may be considered to descend uniformly, but to be increased by $4 \pi n c$ per centimetre of coil. A similar apparent reversal of current against potential takes place in a series of cells supplying a galvanic circuit.

Returning to Fig. 8, the solid angle at any point on the axis CO distant x centimetres from the centre O , is readily found to be $2 \pi \left(1 - \frac{x}{\sqrt{x^2 + r^2}} \right)$, r being the radius of the circle AOB , and the cross-section of the wire being so small that the current may be considered filamentary without practical error. The potential is therefore

$$P = 2 \pi c \left(1 - \frac{x}{\sqrt{x^2 + r^2}} \right).$$

The gradient of this or $-\frac{dP}{dx}$ is $\frac{2\pi C r^2}{(x^2+r^2)^{\frac{3}{2}}}$, and numerically equals the intensity B at any point on the line COB . At the centre O where x is 0, B becomes $\frac{2\pi c}{r}$. A ring set in the magnetic meridian, and employed as a tangent galvanometer superposes on the earth's intensity in the line AOB , the intensity $\frac{2\pi c}{r}$ at O directed along COB , and a small magnetic needle placed there will set its magnetic axis along the resultant of these two rectangular fluxes. If θ be the angle of deviation we must have

$$\frac{\text{coil's intensity}}{\text{earth's intensity}} = \tan \theta = \frac{\frac{2\pi c}{r}}{H},$$

H being the earth's horizontal intensity; from which $c = \frac{H r \tan \theta}{2\pi}$ the usual tangent galvanometer formula.

The solid angle and potential of the ring cannot be so simply expressed for a point not situated on the axis of the ring. Although there are several methods for evaluating them, all involve the use of convergent series whose terms have to be summed until the desired approximation is attained.¹ It is evident from the figure, however, that on the median plane AOB , within the ring, the intensity is a minimum at the centre O for the equipotential surfaces—11 and +11 are furthest apart at that point; yet as regards the axis, B is a maximum at O , for along OC or OB the surfaces separate out as they recede from the median plane.

The total flux threading the wire, or the surface integral of B , is also a complex expression. It increases directly with the linear dimensions of the loop, for by doubling the scale of the diagram, the distance between the equipotential surfaces will everywhere be doubled, halving B everywhere, but quadrupling the area through which it flows, leaving the aggregate flux twice as great.

It is worth passing attention to notice the distribution of flux in a case where there is no system of potential,—the interior of a current-conveying conductor,—as already mentioned. Let R be the radius of a circular wire over whose cross-section AOB , Fig. 13, the current is uniformly distributed. At the axis O , there is no flux, but from this point outwards the flux steadily increases. At any point P ,

1. Professor Perry, Phil. Mag., Dec., '91.

distant r from 0, the current in the external cylindric shell of radii R and r , produces no influence, and the intensity is that which would be produced at the surface of a wire of radius r having the same current density. This intensity from what we have seen on page 9 would be

$$\frac{2}{r} \frac{c}{R^2} \left(\frac{r^2}{R^2} \right) \text{ or } \frac{2}{R^2} c r.$$

The flux takes place in circles all concentric with the axis of the wire. The M. M. F. in any circle of flux is $4 \pi c \left(\frac{r^2}{R^2} \right)$, but this cannot be distributed potentially so that the gradient shall coincide with B .

CHAPTER III.

Reluctance and Permeance.

IN order to determine the flux in a magnetic circuit, we might, in accordance with results thus far obtained, proceed to find the distribution of potentials (assuming that such existed), from this, the gradient B , and then calculate the surface integral of B over an area drawn once across the circuit. But there is another method more direct and comprehensive, which will enable the flux to be found, ideally at least if not practicably, and that is, by determining the total *reluctance* in the circuit. This reluctance, divided into the M. M. F., gives the flux as quotient. Reluctance is thus the analogue in the magnetic circuit, of resistance in the galvanic, with the distinction already referred to in Chapter I, that while resistance checks the flow of current, it exchanges the electrical into thermal energy, while reluctance acts as a regulator of flux only.

We are now enabled to apply Ohm's law to the magnetic circuit in the form $F = \frac{M}{R}$, where F is the flux, M the M. M. F., and R the reluctance of the circuit.

Simple as is the rule, and easy of recollection by its analogy to the fundamental law of the galvanic circuit, it very frequently fails in practical application, owing to the difficulty in detail with the calculation of reluctances. This is incidental to the fact that there is no known magnetic insulator. Resistances in the galvanic circuit are usually easy to determine numerically, for they commonly belong to wires, which are uniform cylinders or are considered as such. Magnetic paths are, on the contrary, almost always divergent and irregular.

The problem of finding the reluctance of a circuit is generally similar to that of finding the joint resistance of a bundle of metallic wires all connected in parallel, whose lengths are variable, and whose individual cross-sections differ from point to point. Apart from this numerical difficulty, the determinations of reluctance and resistance are essentially alike. A substance would have unit specific

resistance or resistivity, when a block of it one centimetre cube, would offer one unit of resistance between two metallic plate electrodes pressed against opposite faces. A M. M. F. of unity, separating two opposed parallel faces one centimetre apart, would establish unit intensity in the intervening space, because the gradient of potential would evidently be one erg on the unit pole per centimetre, for which one c. g. s. line of flux will be accorded to every square centimetre of surface. In other words, unit reluctance is that of one centimetre cube of space in the air-pump vacuum. In reality the difference between the reluctance of space in the exhausted air-pump receiver, and that of wood, copper, flint, and nearly all substances except the magnetic metals, is so small that it may be practically left out of consideration.

As an example of applying the principle of reluctance to magnetic circuits, let us consider a closed circular solenoid of the anchor-ring type, already discussed in Chapter II. Let the cross-section of the ring have any shape, and an area of a square centimetres. Also suppose the mean radius of the ring to be r centimetres, and r to be so large compared with the ring's thickness that the intensity which we know to be greater at the inner and lesser at the outer circles, may be taken as uniform through the ring without sensible error. The winding is to have N turns, and through these a current of c units is to circulate. Then for each centimetre of magnetic circuit, there will be a square centimetres in parallel, giving a reluctance per centimetre

of path $= \frac{1}{a}$, and since the total length of circuit (a circle of radius r) is $2 \pi r$, the total reluctance will be $\frac{2 \pi r}{a}$.

The M. M. F. is 4π times the current turns on the ring, or

$4 \pi N c$, and the flux $= \frac{4 \pi N c a}{2 \pi r} = \frac{2 N c a}{r}$, the result obtained in Chapter II.

As an example, let the ring have a mean radius of 50 cms., and a section of 5 square cms. If uniformly wound with 1,200 turns of thin wire carrying a current of two amperes, or 0.2 c. g. s., the M. M. F. will be

$$4 \pi \times \frac{1}{10} \times 1,200 = 3,016;$$

and the reluctance $\frac{2 \pi \times 50}{5} = 6.2832$.

The flux will thus be $\frac{3,016}{6.2832} = 480$, and the mean intensity

$$B = \frac{480}{5} = 96.$$

Closely related to the conception of reluctance, is another, often very useful, *permeance*.¹ The two are mutually reciprocal. Reluctance having its galvanic analogy in resistance (ohms), permeance represents conductance (mhos). The flux in a circuit is the *m. m. f.* multiplied by the permeance, and if we represent the latter by the symbol *P*,

$$F = \frac{M}{R} = M P; \text{ so that } P = \frac{1}{R}, \text{ or } R = \frac{1}{P}.$$

The permeance of a conducting channel in which the lines of flux are parallel, would be the area of the channel in square centimetres, divided by the length in centimetres; but as such uniform channels are rarely encountered, the permeance has to be found by supposing the actual channels divided into a large number of short columns, comprised between successive equipotential surfaces, determining the permeance of each lamina separately, and afterwards compounding them in series.

As an instance of the application of permeance and its principles, to the determinations of magnetic circuits, we may consider the case of a closed, circular solenoid whose cross-section is too large, relatively to its radii of revolution, for the assumption of a uniform intensity everywhere within. We will suppose that its section is rectangular, as shown in Fig. 14, with sides parallel to the axis of revolution. Let the radii be *R* and *r*, and the height *h*. Also, let the winding layer be so thin that the amount of flux in the substance of the conductor need not be considered. As before, the *m. m. f.* will be $4 \pi N c$. The mean path of the flux will be greater than $2 \pi r$ and less than $2 \pi R$; but it will not be the arithmetical mean of these two limits, because the flux is denser at the inner portions of the section. By dividing up the section into a large number of thin rings such as that represented between the radii ρ and $\rho + d\rho$ we may find the permeance of each separately, and then sum them all for the total permeance. The permeance of the thin ring indicated will be its area $h d\rho$ divided by its length or $2 \pi \rho$, and we have to sum

1. Silvanus P. Thompson, "The Electromagnet," page 173.

all these terms between the radii r and R ; this is given by the integral

$$P = \frac{h}{2\pi} \int_r^R \frac{d\rho}{\rho} = \frac{h}{2\pi} \log_e \frac{R}{r}$$

representing the total permeance.

The flux in the solenoid is

$$F = 4\pi Nc \frac{h}{2\pi} \log_e \frac{R}{r} = 2Nch \log_e \frac{R}{r}$$

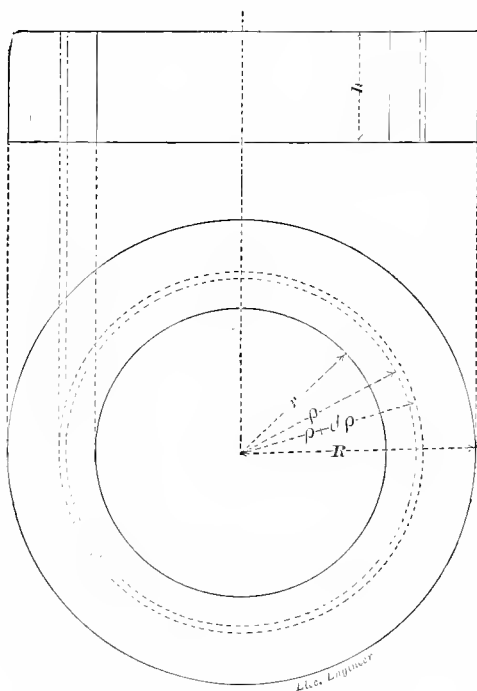


FIG. 14.

This will be the formula to employ for a closed circular solenoid of rectangular section, when the simpler expression already given is not sufficiently accurate for the purpose in view.

As an illustration, let $R = 15$ cms. ; $r = 10$; $h = 5$; and $N = 500$, with $c = 1$ ampere or 0.1 unit ; $M = 4\pi \times 500 \times 0.1 = 628.3$

The permeance

$$P = \frac{5}{6.2832} \times \log_e \frac{15}{10} = 0.3227,$$

so that $F = MP = 628.3 \times 0.3227 = 202.8$ with a mean

$$B \text{ of } \frac{202.8}{25} = 8.112.$$

If we had been content to employ the previous formula, with a mean radius r of 12.5, and a cross-sectional area of 25 sq. cms., we should have obtained

$$F = \frac{2 \times 500 \times 0.1 \times 25}{12.5} = 200,$$

and the mean $B = 8.0$; or we would have underestimated the flux by about $1\frac{1}{2}$ per cent.

Although the difficulties that beset the numerical determination of reluctances are generally prohibitive, the ideas it embodies and suggests are often valuable in dealing with magnetic circuits. Thus it is evident that the flux through a single circular turn of wire carrying nc amperes must be greater than that through a long solenoid of the same diameter, having n turns through which a current of c amperes circulates, for although the M. M. F. in each magnetic circuit will be the same, there will be, roughly speaking, an additional reluctance in the solenoid of $\frac{l}{a}$,

where l is its axial length and a its cross-sectional area, besides that due to a longer exterior path. For a given number of turns, short coils of large area emit the greatest flux. Practice in the measurement of magnetic circuits trains the eye to form an estimate of the flux that a coil will emit, per ampere of circulating current. The conditions for the maximum total flux are often adverse to the development of a high intensity.

Permeances vary directly, and reluctances inversely, with the linear dimensions of a geometrical system of conductors. Suppose that any arrangement of coils or wires is fixed in space, and that a steady current is flowing through them. If we imagine all the equipotential surfaces properly described, the values of the intensity and of the flux in the circuits, or combination of circuits, will be definitely determined. Then if the linear dimensions of the whole system be suddenly increased, or diminished, in a given ratio—if its scale of construction be altered—the permeances of each circuit, or corresponding portion of a circuit, will have been altered in the same ratio. If the scale had been increased three times, the relative arrangement

of equipotential surfaces will not have been affected by the change, but corresponding distances being everywhere trebled, the gradient of potential will be three times less. The lessened intensity will, however, be carried through areas that will be all correspondingly increased nine times, and the aggregate flux will therefore be three times as great as in the original. Or, from the point of view suggested by the conception of permeance, each elementary path of flux will have been lengthened three times, but enlarged nine times in cross-section, so that its reluctance will be three times less and its permeance three times more.

Consequently, if the flux is once determined for a magnetic circuit, it increases directly with the dimensions or scale. This implies that the diameter of the conductors is altered in the same proportion. With this assumption, the flux in a circuit of a given form, number of turns, and current carried, will be proportional to the length of wire it contains.¹ The influence of iron will be subsequently considered.

1. Vide a paper by Carl Hering before the Franklin Institute, May, 1892.

CHAPTER IV.

The Influence of Iron in and on the Magnetic Circuit.

WE have thus far confined our attention to magnetic circuits devoid of iron, circuits surrounding electric currents, and far removed from any of the magnetic metals. We have seen that in all such circuits, there will be a distribution of potential wherever there is a flux, except within the substance of the active conductors; that this potential and flux can always be determined by rule, however difficult the numerical calculation may happen to be, so that the elements of the circuits, such as we have defined them, namely M. M. F. (including potentials), flux, flux density, reluctance, and permeance, are all essentially determinate quantities, depending only on the strengths of the currents, and their geometrical arrangement. We now proceed to consider the effects of introducing iron into magnetic circuits. It will be seen that the changes thus effected are not fundamental but superstructural. The iron alters in no way the original system, but superposes upon it a new system, which the original one has induced, and these two, merged together, form a composite system whose resultant effects are presented to view.

The first evident effect following the introduction of iron into a circuit is an increase in the flux. In order to examine this under the simplest conditions, we will select that geometrical system which we know establishes a uniform intensity, and in a direction of uniform curvature—a closed circular solenoid—with a mean radius r and a small cross-sectional area a . Suppose this to be constructed of wood, and wound with Nc current turns. Then by the results of Chapters IV and VI we may consider that we have every-

where within the wood an intensity of $\frac{2 Nc}{r}$, which we may call the prime intensity. Now suppose the wooden ring exchanged for one of iron having the same dimensions. We should find experimentally that the flux, and consequently the intensity, had everywhere increased in some ratio, μ , whose magnitude would depend on the quality of the iron, the value of the prime intensity, and the manner

in which that prime intensity had been brought to bear upon the iron. But leaving these influences for later consideration, we should find that the flux had increased from F to μF . There would be no other magnetic change. Physically, of course, the magnetic structure and molecular condition of the iron must be different from those in wood, but so far as the magnetic circuit and its various elements are concerned, precisely the same effects would be produced within the wood, if the excitation on it had been increased from Nc to μNc current-turns. ($10 Nc$ to $10 \mu Nc$ ampere-turns.) There are only two ways in which the flux in this circuit can have been augmented by the substitution of iron. Either by an increase in the permeance, the M. M. F. remaining constant, or by an increase in M. M. F. with the prime permeance unchanged.

Here as well as in the corresponding case for the galvanic circuit, purely numerical considerations afford no choice between the two alternative hypotheses. The electrical resistance of a wire might just as well be considered as due to the effect of a counter E. M. F. produced in a definite relation to the current strength. All that we can observe is a fall of pressure along the wire, and this might be ascribed either to opposing E. M. F. e , or to its numerical equivalent cr . There are cases, such as that existing in the arc lamp, where the drop in pressure, largely due to counter E. M. F., was long ascribed entirely to resistance, and a distinction can only be effected in these instances by observing that the drop in pressure is not uniform in distance, or that residual E. M. F. survives the working current. For conductors in general, however, the principle of resistance is much simpler than that of its numerically equivalent counter E. M. F., and its practical advantage is decisive.

With the magnetic circuit of iron it is otherwise, for, assuming the change in flux to be due entirely to diminished reluctance, the reluctance of iron is not constant as is the resistance of metals in the galvanic circuit at uniform temperature, but varies with the conditions of the circuit; while on the other hand, it is found that after the current has been interrupted in the solenoid, a residual flux exists in the iron. If we are to retain the definition that M. M. F. and flux stand in the relation of cause and effect, it follows that there is residual M. M. F. in the iron. It is therefore more probable that the increase in flux is due to M. M. F. induced in the iron and coinciding with the prime M. M. F. in direction. Since the maintenance of flux does not imply

expenditure of power, this does not violate any law connected with the conservation of energy.

According to these principles, the uniform prime intensity of flux in the solenoid, excites in the iron, when introduced, a uniform M. M. F. which will exist in every centimetre of iron under stress along the circuntal path. Since the prime M. M. F. has increased μ times, the induced M. M. F. in the iron must be $\mu M - M = M(\mu - 1)$. Similarly

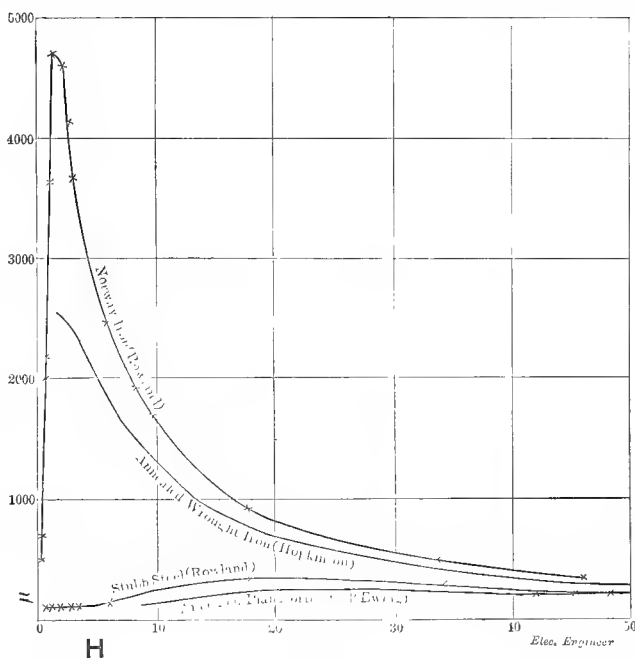


FIG. 15.

the difference of potential primarily existing between any two planes drawn across the ring will have increased μ times as though the original P. D. of $\frac{2 Nc}{r}$ per cm. had been increased by an induced P. D. of $\frac{2 Nc}{r} (\mu - 1)$.

There are thus two complete and associated systems of P. D. flux, and intensity, in the iron ring. The first is the

prime system that would exist if the ring were of non-magnetic material. The second is the induced system $\mu-1$ times greater. On interrupting the current in the solenoid, and thus withdrawing the prime system, this induced M. M. F., flux and intensity remain. Strictly speaking the induced system does not remain complete, but suffers to a greater or less extent when the excitation is withdrawn, but under favorable conditions, as much as 95 per cent. may be conserved.

The numerical value of μ , the factor of augmentation due to the presence of iron depends upon the chemical and physical structure of the latter. For a given prime intensity, it is greater as the iron is softer and more nearly pure, while it is materially reduced by small degrees of impurity, especially when these consist of carbon or manganese. A value as high as 4,600¹ has been recorded for Norway iron in a particular case, and on the other hand a result as low as 1.85 has been reached under very exceptional conditions.² Fig. 15 gives some curves of values for μ with several qualities of iron. The abscissæ represent prime intensities, or the flux per square centimetre that would exist if the iron were removed and replaced by non-magnetic material. We know that this is the gradient of potential, and that in the circular solenoid it corresponds to the uniform fall of potential per centimetre of circuit. Prime intensity is commonly denoted by the symbol H . For example, in Norway iron, μ is seen to be 1,000 for about 16 c. g. s. lines per sq. cm. ($H=16$). If this particular intensity were produced in the interior of a closed circular solenoid, the complete introduction of Norway iron of this quality into the core would raise the intensity to 16,000 lines per sq. cm. and the P. D. per centimetre of circuit would be raised from 16 to 16,000 ergs per unit-pole. If the circumference of the ring were 200 cms., the exciting M. M. F. would have been 3,200, and the total M. M. F. after introducing the iron 3,200,000.

If we next consider circuits partly composed of non-magnetic material, and partly of iron, we have to add one more link to the chain of consequences, and take into consideration the demagnetizing influence of the iron on itself. The M. M. F. induced in the iron establishes its own local magnetic circuit independent of the prime system, and its flux on the returning path that completes this local circuit,

1. Rowland. Phil., May, 1873.

2. Ewing. "Magnetism in Iron," page 130.

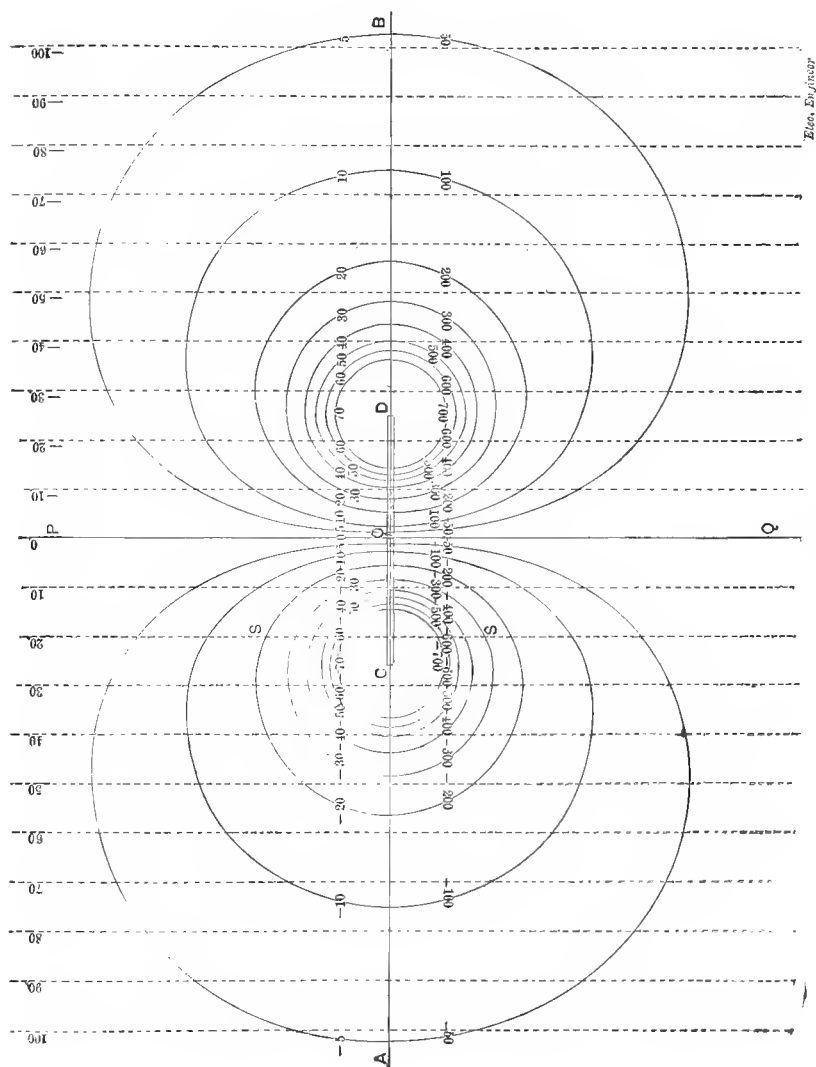


FIG. 16.

entering the iron in the reversed direction, tends to neutralize the excitation of the prime flux. The induced M. M. F. reached in the iron at each point is such that it is just $\mu - 1$ times the resultant of the prime and induced counterflux.

Figs. 16 and 17 represent the effect of placing in a uniform flux, a soft-iron bar 50 cms. long, and 20π square cms. in cross-section. The prime distribution of equipotential parallel planes is indicated by the dotted lines 10 cms. apart, and since the gradient is 10 ergs-per-unit-pole in this distance, $B = 1$, from left to right. The bar C O B is now introduced with its axis parallel to B . If the value of μ for the iron composing it, with $H = 1$, happened to be 2001, the induced distribution of potential would be approximately that indicated by the closed curves to the scale of inverted numbers. The external potentials are indicated only from $+700$ to -700 . By rotating the diagram about the axis A O B, these curves will generate the corresponding equipotential surfaces. Within the bar the equipotential surfaces would be planes perpendicular to the axis with $B = 2001$ from left to right. Outside the bar the flux completes the local magnetic circuit of the induced system by curved diverging paths from right to left.

The induced system would be similar to the straight solenoid or helix in Fig. 4 discussed in Chapter II, page 12, and the bar with this distribution could be replaced by a solenoid of the same dimensions without iron, and having

$$\frac{2000}{4\pi} \text{ current-turns per cm. or } \frac{100000}{4\pi} \text{ current-turns in}$$

all. But the bar could never attain this distribution, owing to the counteracting influence of its own external return flux. There is no magnetic insulator which will keep this return flux outside the bar, and it penetrates the iron with an influence equal to that of prime flux with the same intensity. Thus where the surfaces of 700 and 300 ergs cut the bar, they are rather less than 5 cms. apart. The mean

value of B in this length will be $\frac{700 - 300}{5}$, or 80, due

to the external flux, or 80 times more powerful than the prime flux to which it is opposed. This demagnetizing influence would be most active near the extremities of the bar, where the external fall of potential is most rapid, and least near the centre. The result will be that the flux induced in the bar instead of taking the value $H(\mu - 1)$ which we know it would assume if there were no counter-

acting return flux—that is, if the bar formed the core of a closed circular solenoid—rises to the value at which it is $\mu - 1$ times the local H or resultant of the prime and induced distributions. Even if the prime flux be uniform, the induced return flux will generally vary from point to point, and the value of μ corresponding to their resultant or combination, will also be in general a variable, so that

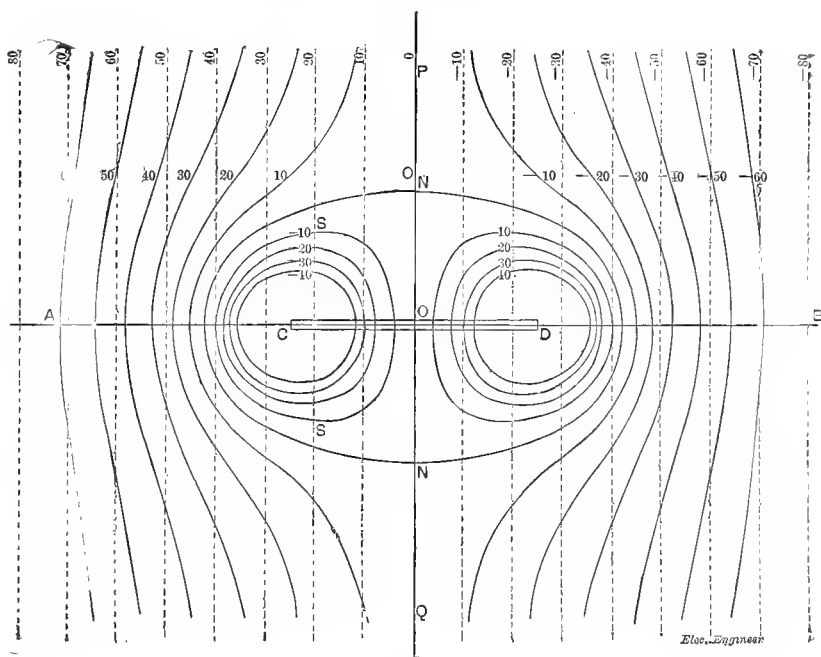


FIG. 17.

the induced system in the iron usually becomes very complex.

In the case of Fig. 16, we may, for the sake of illustration, waive all attempts at accurately depicting the resulting induced system left in the bar under the combined influences of the prime and its own return fluxes. We may assume that the relation of μ to varying values of H , as determined by experiment, may be such that the modi-

fied potential is everywhere 10 times less than the unmodified potential would have been, in which case the same distribution of equipotential surfaces here indicated will apply, the scale being altered to the upper numbers. This is the final condition of the induced system, and is independent of, although superposed upon, the prime system. If we add together at all suitable points the potentials of the prime and final induced systems, we obtain a new series of equipotential surfaces represented in Fig. 17. For instance, the points *ss* situated on the prime plane of $+20$ and the induced surface of -30 will have a resultant potential of -10 , and the surface of -10 in the resulting system passes through them. Here the dotted lines still indicate the prime equipotential planes, and the curved lines the resultant system.

Notwithstanding the arbitrary manner in which we have simplified and chosen the final induced system, this diagram fairly represents the general effect of introducing a bar magnet or rod of soft-iron into a uniform field. Broadly, the prime equipotential surfaces are displaced, as an elastic medium might be, by the introduction of a wedge. The flux is intensified along the line of the axis, and weakened in the neighborhood of the equatorial plane. The surface of zero potential has two sheets, one in the equatorial plane, and the other an ellipsoid enveloping the bar. At their intersection *NN* a neutral circle is formed, obtained by revolving the radius *ON* round the axis *AOB*. At all points on this circle the flux vanishes, and a small compass needle would not assume any selective direction. That is to say, on this circle, the prime flux from left to right just balances in intensity the return flux of the local or bar circuit, from right to left.

In the above instance we have considered the case of a long, straight and narrow bar placed lengthwise in a uniform flux; but the removal of these restrictions, while it may add numerical complexity, brings no additional intricacy of process. In all cases we have first to determine, from the currents and their geometrical arrangement, the elements of the prime system with the iron removed. Then introducing the iron into the circuit, the final induced system has to be found, such that at every point the induced *M. M. F.* is $(\mu - 1)$ times the value of the resultant intensity due to prime and induced distributions, μ being a function of the intensity experimentally determined. We might consider the iron as divided up into bundles of rods arranged everywhere parallel to the flux,

and find the distribution round one rod at a time as though that existed alone, and then superpose the successive distributions from neighboring rods, extending the process until every rod had its influence compounded. Or it might be possible to deal with the iron in mass by some more comprehensive method that would render this long summation unnecessary; but the final induced system, however arrived at, when superposed upon the prime distribution would yield the resultant system actually developed in the circuit.

On the removal of the prime system, the induced system is left active, but the iron, no longer submitted to the prime excitation is exposed solely to its own return flux tending to reduce the residual *M. M. F.* The demagnetizing influence will vary from point to point, depending upon the distribution of potential and flux in the induced system; but wherever the demagnetizing flux exceeds a certain intensity depending upon the quality of the iron, a process of decadence will set in. The induced system may be destroyed in a small fraction of a second, or it may be sustained nearly complete for an indefinite period according to the range of μ in the iron, and the form of its circuit, which latter regulates the intensity of the demagnetizing return flux.

These conditions of magnetization in iron are readily apprehended qualitatively, but they are numerically so complex that the elements of a magnetic circuit partly composed of iron, cannot in general be computed. They have in fact only been determined in a few simple cases, and usually on the assumption that μ has a constant value.

It is evident that from the standpoint of magnetic circuits, a magnet is equivalent to a particular form of active solenoid. A piece of magnetized iron is physically very different from a conductor carrying a current, but magnetically it is a particular aggregation of small magnetomotive forces and as such the corresponding arrangement of conducting loops must always exist.

Ewing's theory is the most recent and comprehensive hypothesis accounting for the magnetic behavior of iron. According to this view, every molecule of iron may be regarded as an elementary independent magnet, free to set itself in any direction under the influences of magnetic flux. Each molecule has a separate magnetic circuit, in which the *M. M. F.* is inherent and fixed. The flux from any one molecular magnet, while extending over the whole system,

will be practically confined in its action to those molecules in closest proximity. The result of all the mutual actions will be an aggregation of magnetic circuits in all directions and degrees of complexity, so that the resultant *M. M. F.* in any one line is zero. The iron is then neutral. If a gradually increasing flux is brought to bear upon the system from an exterior *M. M. F.*, at first only a few molecules in balanced or unstable equilibrium will be affected and turned into alignment with the flux. Gradually molecules in firmer mutual association will be separated from the configurations they had naturally assumed, and brought into more or less direct alignment with the impressed flux. There will be a certain strength of flux which will suffice to destroy a large majority of all the groups. At this stage μ will reach a maximum value. Beyond this point only the more powerful and enduring arrangements will remain to be reduced, as well as the more perfect alignment of those already mastered, while a few of the most powerfully united groups will require comparatively great intensities to resolve them. At an indefinitely great intensity, every molecule will be brought into parallelism. In this condition, the *M. M. F.* induced in the iron will have reached its maximum, and be equal to the sum of all the molecular *M. M. F.*'s in any one line. At any intermediate stage, the induced *M. M. F.* would be the sum of the *M. M. F.*'s now more or less dispersed, resolved along this line, the components at right angles to the flux disappearing in the aggregate by cancellation.

Besides offering explanations for a variety of magnetic phenomena, this theory accounts for the observation that μ does not merely depend upon the magnitude of the prime intensity *H*, but also upon the manner in which this magnitude has been reached; for the molecular disengagements will generally differ if this value be attained by steady increase, by reversals, or again by overreaching and descent, so that the condition at any moment involves the previous history of the iron, or gives evidence of what has been termed its "magnetic memory." We shall return to this subject at a later period.

In most electromagnetic machines, the magnetic circuit is principally of iron, and consequently approximates to the type of a closed solenoid with iron core. This greatly facilitates the computations, for although there is some return flux, whose influence prevents an accurate determination from being arrived at, unless experimentally, yet the demagnetizing flux is generally small compared with

the prime excitation, and a sufficiently close approximation to the resulting magnetic circuit is usually available to meet practical requirements. It is only when the circuit has a lesser proportion of iron and larger air-gaps, that the numerical difficulties in determining the resulting system become formidable.

CHAPTER V.

Stress Distribution.

THE flux in a magnetic circuit is accompanied by a condition of stress. In a perfect fluid at rest, the stress at any point is equal in all directions. Magnetic stress, however, varies from a negative value or tension in the direction of the flux, to a positive value or pressure in the perpendicular plane. Assuming then that ether is a perfect fluid, this observed condition is an argument for the existence in it of motion for which flux is the origin, associate, or result. This is the basis of Maxwell's theory of molecular vortices.

Whatever may be the ultimate cause of this stress, all the observed moving forces that are exerted between currents or magnets may be attributed to its existence. If we consider any point in the magnetic circuit where the intensity is B , the stress will be represented by a pressure of $\frac{B^2}{8\pi}$ dynes per square centimetre in all directions, like a simple fluid pressure, with a superimposed tension of $\frac{B^2}{4\pi}$ dynes per square centimetre in the direction of B . The resultant pull in this direction will therefore be a tension of $\frac{B^2}{8\pi}$, and, broadly speaking, the lines of flux tend to shorten themselves, while repelling each other sidewise. This is the fundamental moving power in nearly all electrodynamic machines.

Strictly speaking, all substances appear to have some capability for magnetization. The effect is ordinarily so feeble that it can only be discovered by placing two substances in a powerful flux, so that the difference of their magnetic properties may enable them to exert a relative displacement. Excluding therefore the magnetic metals and their principal chemical derivatives, matter is apparently unaffected by permeating magnetic flux, and the stress may be said to exert as negligibly little disturbing influence upon the substances in its path, as hydrostatic stress might effect upon a sponge immersed in the cylinder of a hydraulic press.

Considering the case of a long straight wire at rest, far removed from all magnetomotive forces, it is evident that the stress set up when a current passes through the wire cannot tend to displace it bodily, for we know that the flux, and consequently the stress, is symmetrically distributed in concentric cylinders round the axis. There will be a uniform pressure on the surface of the wire tending to compress it and reduce its diameter, keeping the axis at rest. But any flux from neighboring magnetomotive forces, or

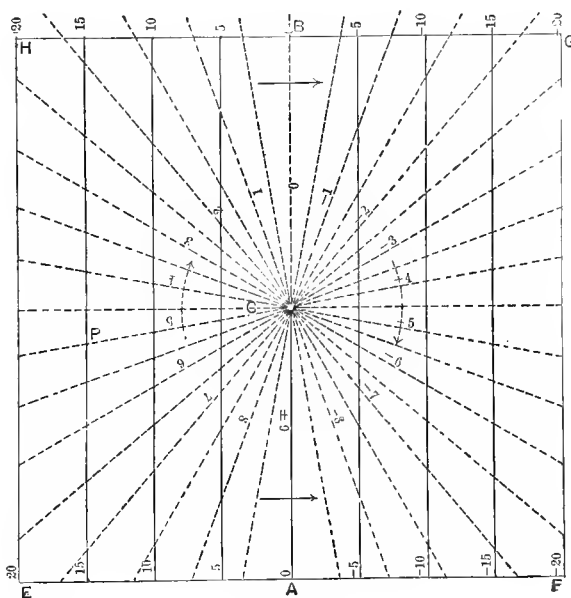


FIG. 18.

from portions of its own circuit, not in its own straight line, will in general disturb this symmetry, and give the wire a tendency to move.

This pressure in the case of a wire of radius r carrying a current c will be $\frac{2 c^2}{\pi r^3}$ dynes per cubic centimetre of the wire's substance. Thus a straight wire carrying 30 amperes and having a radius of one millimetre would have a

pressure exerted on its surface due to the stress from its own flux of $\frac{2 \times 3 \times 3}{3.1416 \times (0.1)^3} = \frac{18,000}{3.1416} = 5,730$ dynes per cubic centimetre of its volume, and a small element in or near the surface of any shape occupying $\frac{1}{1000}$ th of a cubic centimetre would be forced towards the axis by a pull of 5.73 dynes. Three-fourths of this stress is due in a round

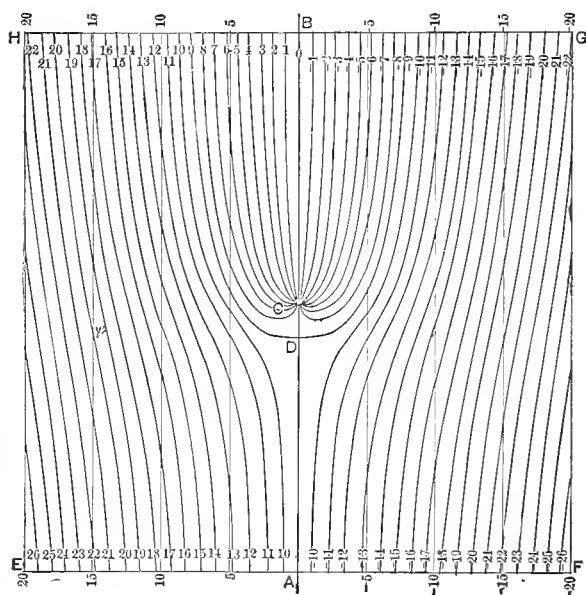


FIG. 19.

wire to side pressure and one-fourth to the tension along flux paths encircling the surface.

Fig. 18 represents two systems of potential. One, indicated by the parallel straight lines, belongs to a uniform intensity. The other, shown in radial dotted lines, is due to a long, straight, current-carrying wire appearing in section at c. If the parallel lines are one centimetre apart $B = 5$ in the first system over the 64 square centimetres of space included in the diagram. Also since $4 \pi c = 18$, the current establishing the second system has a strength of 1.432 or 14.32 amperes.

For numerical convenience, the cyclic elevation of potential round the wire is indicated as taking place at the line $A O B$ in the surface of zero potential for the first system. The arrows indicate the respective directions of the flux in the two systems and are assumed to lie in the plane of the paper. The current c must therefore be flowing down the wire from the observer, since the nut on a right-handed screw rotating with the arrows would move in that direction.

If these two systems coexist, their effects will necessarily be combined. Fig. 19 gives the resulting system, the curves representing the sections in the plane perpendicular to the wire, of the equipotential surfaces surrounding a current of 14.32 amperes in a uniform prime intensity of 5. The faint straight lines indicate the surfaces of the first system. The point r for example, which stood in Fig. 18 on the surfaces of $+15$ and $+5$, now resides on that of $+20$. The general effect is a crowding of the contours between B and C , with a more sparse distribution between C and A . The intensity is increased above C at the expense of the region below. The potential is numerically discontinuous at the line $C A D$, but the gradient is uniform on the sides of the gap so that there is no discontinuity in B .

At the point D , the perpendicular intersection of two equal surfaces there is absolute neutrality, and here the fluxes from the two systems, equal and opposite, disappear in union. The distance of this point from the centre of the wire we know to be $\frac{2c}{B}$, or, in this instance, 0.573 cm. Generally if B represents the prime intensity, and C the vector current, the resultant intensity B^1 at any point whose axis vector is ρ in the plane outside the wire will be defined by the relation

$$B^1 = B - \frac{2}{\rho^2} V. C \rho.$$

The combined influences of the tension along the lines of flux that bend over the wire, and the side pressure from the more crowded flux above, are to press the wire down from C towards A with the force expressed in dynes by the equation

$$P = V. C B;$$

and this relation holds in all cases for a steady, straight current C making any angle with a uniform intensity B . In Fig. 19, B and C are mutually perpendicular, their

influence is a maximum, and $P = 1.432 \times 5 = 7.16$ dynes acting along CA , upon every centimetre of the wire's length. In 3 metres of this wire, the total magnetic pull would amount to 2,148 dynes or 2.19 grammes weight.

The simple equation last stated is practically important. It shows that when the straight wire is not perpendicular to the flux, but is inclined to this at an angle θ , the pull is reduced to $CB \sin \theta$ dynes' per cm. The wire may be considered in such a case as having two components, one $\cos \theta$ along the direction of flux, and uninfluenced by

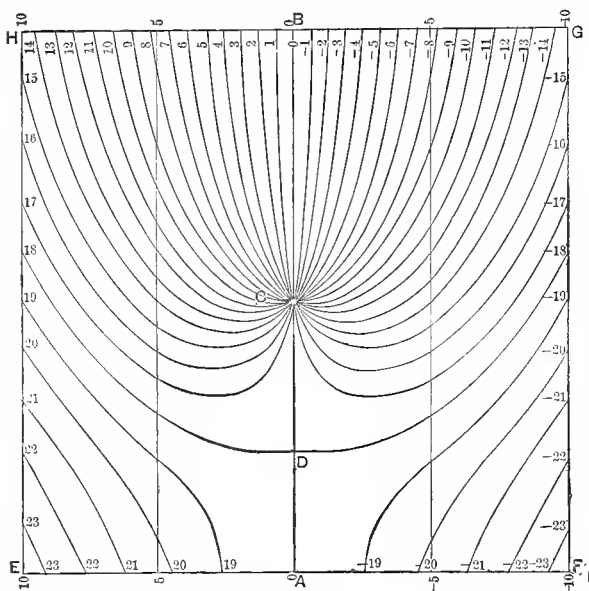


FIG. 20.

stress; the other $\sin \theta$ across the flux perpendicularly. From this point of view, the whole pull is exerted upon the virtual component of the wire's length at right angles to the flux. Again, if the wire is not straight, or the flux not uniform, it becomes necessary to subdivide the length into such a number of smaller sections that the wire may be considered straight and the field uniform in these narrowed limits without sensible error. The pull must then be found for each section separately, and the resultant of all the actions

determined. In some cases it will be possible to avoid this tedious process by finding an average by inspection; in others the rules of the integral calculus will be applicable. But however complex the arrangement in the number of wires or the difference of strengths of current they carry, the fundamental principle is the same,—the pull on each element dl of conductor carrying a current c in an intensity B making an angle θ with its direction, will be $dl \cdot c \cdot B \sin\theta$ dynes directed perpendicularly to the local plane of B and C ; and each element acts independently of all the rest. The direction of motion is perhaps best remembered by Fleming's rule, often useful in dealing with motors. If the *first finger* of the left hand points in the direction of *flux* (that is by following the north-seeking end of a compass needle), and the bent middle finger, the current,—the outstretched thumb perpendicular to these will point out the direction of *motion*.

We have hitherto assumed that the prime flux was not altered by but merely compounded with, the flux due to the currents in the wire or wires. If, however, the flux of the currents alters the prime distribution, as might take place in the vicinity of iron by the establishment of induced M. M. F.'s, these alterations would have to be accounted for and effected in B , if they are of sufficient importance to affect the result, in order to arrive at the true moving forces.

Fig. 20 gives the resultant potential system for a straight wire with $C = 2.864$, and uniform prime intensity $B = 2.5$. P has the same value as in the preceding case of Fig. 19, the prime flux being halved, and the current doubled. It is to be observed that notwithstanding the equality of forces in the two instances, the distributions of potential and therefore of flux differ materially.

At every point of a magnetic circuit there exists, as we have seen, a tension of $\frac{B^2}{8\pi}$ dynes per square centimetre, normal to the equipotential surfaces, and also a lateral pressure of $\frac{B^2}{8\pi}$ dynes per square centimetre acting uniformly in all directions over the tangent plane, the plane with which the equipotential surface at the point locally coincides. Also at every point not occupied by a source of M. M. F., that is, at every point not within the substance of a magnet or active conductor, this distribution of stress is in statical equilibrium, and it would seem that this distribution (or some numerical multiple of the same) is the

the radii OC and OD (including an angle $d\theta$) and the circles AEE and CDG whose radii are numerically r and $r + dr$, respectively. The faces AC and BD are equipotential surfaces. Their area is $1 \text{ (cm.)} \times dr$, and the intensity by the results of Chap. II, $B = \frac{2C}{r}$, is perpendicular to

them. Considering the element as a rigid body, it will be subjected to pressures on the faces AB and CD , together with tensions across AC and BD . These four stresses may be considered as exerted at the centres of the respective faces and are indicated by the arrows. They may be reckoned as follows :

Face.	Force.	Direction.	Theoretical value per sq. cm. dynes.	Local value of B .	Local value of $\frac{B^2}{8\pi}$	Area of face.	Total force acting on face.
AB	Pressure	SP	$\frac{B^2}{8\pi}$	$\frac{2C}{r}$	$\frac{4C^2}{8\pi r^2}$	$r d\theta$	$\frac{C^2 d\theta}{2\pi r}$
CD	"	TP	"	$\frac{2C}{r+dr}$	$\frac{4C^2}{8\pi (r+dr)^2}$	$d\theta(r+dr)$	$\frac{2\pi (r+dr)}{C^2 dr}$
A	Tension	PQ	"	$\frac{r+dr}{2C}$	$\frac{8\pi \left(r+\frac{dr}{2}\right)^2}{4C^2}$	dr	$\frac{2\pi \left(r+\frac{dr}{2}\right)^2}{C^2 dr}$
BD	"	R	"	$\frac{r+\frac{dr}{2}}{2C}$	$\frac{8\pi \left(r+\frac{dr}{2}\right)^2}{4C^2}$	dr	$\frac{2\pi \left(r+\frac{dr}{2}\right)^2}{C^2 dr}$

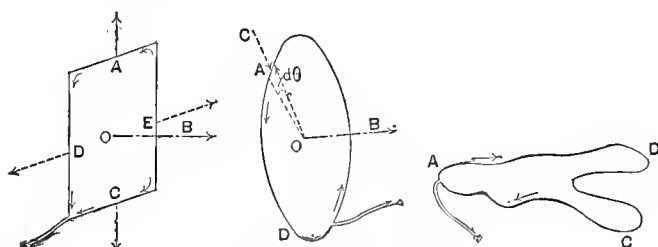
The table shows that there will be a difference of pressure on the inner and outer faces of $\frac{C^2 d\theta dr}{2\pi r(r+dr)}$, or ultimately $\frac{C^2 d\theta dr}{2\pi r^2}$ dynes in the direction *SP*, because the flux is stronger on the inner side, and the stress varying as its square, more than compensates for the difference in area. On the other hand, the tensions are seen to be numerically equal, but they are not exactly opposed, being inclined to each other at an angle $\pi - d\theta$. The effect of this inclination will be a resultant pull of $\frac{C^2 d\theta dr}{2\pi \left(r + \frac{dr}{2}\right)^2}$ or ultimately $\frac{C^2 d\theta dr}{2\pi r^2}$ dynes inwards in the direction *TP*, just counterbalancing the resultant side thrust. The element is therefore in stable equilibrium, and since the same could be proved for any similar small cylindrical element outside the wire, it must follow that any larger portion of the space, which could certainly be built up of such adjacent elements, would be in similar equilibrium regarded as a whole.

The proposition arrived at in the foregoing simple case is of general application no matter how complex or irregular the flux distribution may be, and may be stated in the following terms. The resultant side thrust that may be exerted upon any element of the magnetic circuit (not included in the substance of a magnetomotive source) due to the variation of the intensity *B* in or round it, is balanced by the resultant tension in the opposite direction owing to the local curvature in the flux. In fact uniform flux is straight, while variable intensity always bends so as to keep convex on the weaker side. In other words, the divergence of equipotential surfaces implies not only weakening but also bending in the flux channels, the two effects everywhere maintaining statical equilibrium.

We observe therefore, that when an inactive conducting wire is situated in a prime flux density *B*, the stress distribution is in equilibrium throughout the field, in the substance of this wire as well as all round it. As soon as a current *C* is established in this conductor, the flux distribution from the current will have been superposed upon the prime system. The resultant system of stress distribution will again be in equilibrium at all points outside the wire,

but there will no longer be equilibrium in the space occupied by the wire.

Regarding the wire as a mere filament without sensible cross-section we have seen that the resultant vector force acting upon it will be $P = V. CB$, and this force must be applied to the wire to restore equilibrium between its substance and its environment. When the cross-section of the wire cannot be fairly considered negligible, it may be divided into filaments, each carrying its appropriate share of C . The consequence of applying the rule $P = V. CB$ to each filament in succession, will be that B the prime intensity for any one filament, will be modified by the flux from all the remaining filaments. The vector force on the wire as a whole will still by summation be $P = V. CB$ as before, where B is the prime intensity for $C = 0$; but there will now be a system of mutual forces between the fila-



FIGS. 22, 23 AND 24.

ments in addition. These filaments in fact attract each other, and the resultant forces of the mutual system will depend upon the curvature of the wire, as well as upon the dimensions of its cross-section. For a long straight wire of circular section, with a current density of σ units (c. g. s.) per square centimetre, the force on any element of the conductor at radius vector r will be $P = 2\pi R \sigma^2$ dynes per cubic centimetre, agreeing with the force at the surface noticed on page 45. Practically, the external resultant $V. CB$ is simple and important, while the internal system of forces through the substance of the wire is more complex, and rarely needed.

It may be advantageous to analyze a few simple systems of electromagnetic force distribution. Fig. 22 represents a wire $ADCE$ bent into a square, the length of one side being, say, S centimetres. The square is supplied by two wires held very close together, with a current C ;

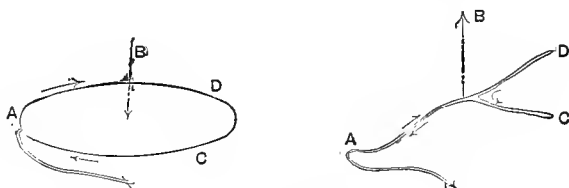
and its plane is perpendicular to a uniform prime intensity B represented by the arrow OB . Viewed on this side, the current circulates counter-clock-wise. The electromagnetic force acting on each side of the square due to B and C will be SCB dynes, represented by the arrows at A , D , C , and E , in the same plane.

These will not be the only forces acting on the wire, for should the prime flux B be withdrawn, there would still be a magnetic circuit having its $M. M. F.$ in the turn of current and its flux threading the loop in the same direction as the prime flux OB . Although the distribution of this local flux would not be uniform over the square, the resultant effect will be to establish forces on the sides in the direction of the arrows. The complete system of forces will then be triple. First a compression of the wire towards its axis due to filamentary attractions; second, a set of forces acting outwards and perpendicular to each side, the action varying in intensity from point to point since the flux distribution is not uniform; and third, a force SCB acting on the establishment of B from the external $M. M. F.$, which force is perfectly uniform, namely: CB dynes per unit of length along any side. Of these three systems, the first two are evidently not reversible in direction when the current in the square is reversed, because the two factors, local flux and local current reverse together. The third system is however reversed by a change in the direction of current, or external flux B .

In Fig. 23, a circle ADB is substituted for the square of Fig. 22. Here the force on any small arc $r d\theta$ due to the action of B and C is $r d\theta CB$, and according to the direction of current shown it will in this instance be directed towards the centre. Summing all such elements, the total resolved force across any diameter will be $2rCB$ and half this, or rCB will be the pressure that the wire must at every point longitudinally support. The triple system of forces will again be: A compression of the wire; a radial thrust of all the parts of the circle outwards due to the application of the rule $V. CB$ for its own local flux, (in this case a symmetrical stress system), and, third, a thrust radially outwards, or inwards, bringing a longitudinal stress of rCB dynes on the wire according to the direction of the external flux B . In this case it will be possible for the second and third systems to oppose and exactly neutralize each other.

As a more general case, suppose a loop of perfectly flexible wire ABC , Fig. 24, to lie in a horizontal plane and to be

capable of moving only in this plane. This condition eliminates any disturbance due to gravitation. On passing a current in either direction through the loop, the forces set up will expand this, until equilibrium is found in a circular form, and further expansion is resisted by longitudinal tension. This expansive tendency will be further assisted by an external flux vertically downwards, as shown in Fig. 25. If, on the contrary, a prime flux had been established upwards through the loop before the current passed in the direction of the arrows, the loop would have tended to collapse and assume some such form as that in



FIGS. 25 AND 26.

Fig. 26. Total collapse will be resisted by the local magnetic circuit and its outward stress system. The equilibrium would in any event be unstable, and if free to move, the sides of the loop could cross each other until the wire opened out into a circle like that in Fig. 25, but with a cross at A, and with the direction of current opposite to that indicated.

This triple system of forces forms perhaps the simplest analysis in these instances, but it would, of course, be possible to arrive at the same results by determining the resultant system of flux in and outside the loop due to its own M. M. F., and that of the external source, followed by a calculation of stresses on the conductor from point to point by the fundamental laws of tension and side thrust.

As a case next in order of complexity, we may consider a wire bent into the form of a square with length of side l , but so situated that the normal to its plane makes an angle θ with the direction of a uniform field of intensity B . This differs from the case last considered only in the departure of θ from 90° .

Let B be withdrawn, and a current of uniform strength C pass steadily through the wire. The compression on the wire's substance, and the stress acting on the sides of the square, tending to force them outward—systems 1 and 2,

according to the results of the preceding case—are evidently unaffected by the direction of the plane assumed by the square in space, these forces being only dependent on the distribution of current.

Now introduce the external flux B , whose direction, indicated by the arrow OB makes an angle θ with the plane normal ON , Fig. 27, drawn positively, or upwards from the plane, on that side which presents the current as flowing counter-clockwise, and its magnetic potentials therefore positive. We may first assume that θ is less than 90° .

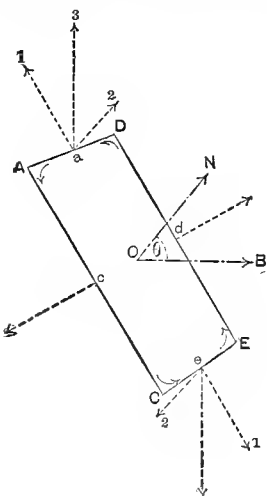


FIG. 27.

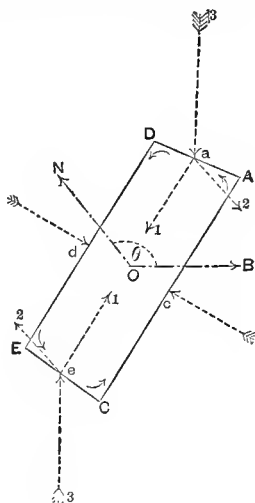


FIG. 28.

By rules already discussed, the resultant force on the sides AC and DE will be the vector products of the currents through those sides into B , or numerically $CB l \cos \theta$ dynes in the plane of the square, as represented by the arrows at c and d . Upon the sides AD and CE , B acts perpendicularly, so that the stress it produces on each is numerically $BC l$ dynes in the directions perpendicular to both wire and B , as shown by the arrows a_3 and e_3 . These forces make an angle θ with the plane of the square, but resolving into components in and normal to this plane, there remain forces $BC l \cos \theta$ dynes acting along a_1 and e_1

outwards, and $B l \sin \theta$ dynes along a_2 and e_2 , tending to rotate the square about an axis in its plane, which by symmetry must connect the middle points $c d$, of $A C$ and $D E$.

These results may also be reached from a slightly altered point of view. B may be resolved into two components, one $B \cos \theta$ normal to the plane, the other $B \sin \theta$ lying in the plane. The resultant stresses produced by $B \cos \theta$ on each side of the square are all directed outwards with side resultants $B C l \cos \theta$. The stresses produced by $B \sin \theta$ are $B C l \cos \theta$ upon the sides $A D$ and $C E$ in the directions e_2, e_2 , and the remaining sides $A C$ and $D E$, parallel with this component, are unaffected by it.

If then the resultant system of flux distribution from C and B were completely mapped out, and the consequent stress distribution in virtue of $\frac{B^2}{8\pi}$ duly determined, there

would be equilibrium at all points everywhere outside the wire, while the effect on and in the wire may be represented by four systems of forces. The first two, pertaining only to C are the above-mentioned compression and outward push in the plane. The third will be $B C l \cos \theta$ on each side, also directed outwards, and the fourth will be a couple of moment $B C l \sin \theta$, tending to rotate the whole square about its central line $c d$.

The direction of rotation would be such as to bring the square perpendicular to B , or in other words diminish the angle θ ; and if the square were actually constructed as a rigid frame pivoted on an axis through $c d$, it would be necessary, excluding the consequences of pivot friction, to apply a couple of $B C l \sin \theta$ dyne-centimetres to the frame to maintain its equilibrium under these conditions.

Fig. 28 represents the similar stress system when the angle θ exceeds 90° , so that the flux from the local circuit is opposed to B within the square. In the equivalent quadruple stress system for this case, the first two components regarded from the normal $o n$ are unchanged, but the last two are reversed. $C B l \cos \theta$ now acts on each side inwards, and $C B l \sin \theta$ acts on $A D$ and $C E$ in the directions of the arrows a_2 and e_2 , forming a couple $C B l \sin \theta$. This couple tends to rotate the plane first into parallelism with B , and then to continue the rotation through the angle θ until the normal $o n$ coincides in direction with B .

We may apply these principles to the case of a rigid,

parallel to the axis and in the direction of the arrow TR ; while with those on the other side the force will follow the opposite direction, that of the arrow VS . At the points H and K , a small element of the wire, ds , meeting $B \sin \theta$ at right angles, will be urged with a force $C B \sin \theta ds$ dynes; on elements at o and q , the force will be nil, and at any intermediate element, such as N , whose position angle NPH is ϕ , the force will be $C B \sin \theta ds \cos \phi$. If all these elementary forces be summed round the circle (whose radius is r), the total force will be $2 C B \sin \theta r$ dynes in each semicircle. This total force applied at the points T and V , whose distances from P are each $\frac{\pi r}{4}$ will be the resultants of the elementary system.

Since each turn of wire will contribute an equal pair of resultants, the total forces referred to the mid-plane of the solenoid will be $2 N C B \sin \theta r$, at distances $\frac{\pi r}{4}$ on each side of P . The moment M of this couple will therefore be $2.2 N C B \sin \theta r \cdot \frac{\pi r}{4} = N C B \sin \theta \pi r^2 = A N C B \sin \theta$, where A is the area of cross section, and if a pulley of radius R were attached to the pivot axis at P , it would be necessary to apply a force of $\frac{M}{R}$ dynes, or $\frac{A N C B \sin \theta}{981 R}$ grammes weight at the periphery of the pulley, in order to maintain the coil's equilibrium, assuming the absence of pivot friction.

To take numerical values that might be readily attainable experimentally, if $N=500$ turns, $B=1,000$ c. g. s., $A=2$ sq. cms., $C=2$ amperes or 0.2 c. g. s., $\theta=30^\circ$, so that $\sin \theta=0.5$, we have for M , $2 \times 500 \times 0.2 \times 1,000 \times 0.5 = 100,000$ centimetre-dynes.

A solenoid in a uniform prime flux, experiences therefore, no tendency to translatory motion, but only to rotation; (a tendency to translation would in general be set up if B were not uniform). This rotatory tendency, or moment M , for a given solenoid, exciting current, and prime flux B increases with the deviating angle θ , varying at its sine, from unity when B lies across the solenoid, to zero when B is in line therewith.

Further the formula for M takes no cognizance of the shape of the solenoid, which is represented simply by the area turns NA . It might be short or long, thick or thin,

the moment would be the same so long as NA remained unaltered. The rotatory power of a solenoid carrying a given current in a uniform prime flux, therefore depends only on its total area turns, and a single turn of 50 sq. cms. will exert the same force as 20 of 2.5 sq. cms. each. These turns need not be all equal of size, provided that they are all concentric; ten in the inner layer might have 2 sq. cms., the outer layer of ten having 3 sq. cms., the sum of the area turns would remain 50. Strictly speaking, this assumes that the wire is filamentary; but if the diameter of the wire is sufficiently great to introduce sensible error, further approximation may be attained by supposing the wire to be subdivided into filaments sharing the current, and summing the area turns again on this basis. The proposition may be true even if the turns be not circular, for it is evident that they may successively vary in size and shape, if each is symmetrical about a line in its own plane perpendicular to the component $B \sin \theta$ and that all the successive median lines lie in one plane, which will be a plane of symmetry for the coil. Whenever such symmetry is preserved, as is usually the case in practice, the moment of rotatory force is $C B \sin \theta$ multiplied by the area turns for the coil.

These principles are characteristic of the behavior of magnets temporary or permanent in a uniform prime flux. They tend to set their own local and internal flux parallel with that in the exterior field, and the tendency to this alignment, the moment of restitution, is proportional to the sine of the deviating angle. A uniformly magnetized rod is in fact equivalent to a solenoid of the same dimensions carrying a number of current turns that will produce the same magnetic potential difference between its terminal planes, as actually exists between the rod's extremities. In practice, magnets are seldom shaped like solenoids, and are never quite uniform in magnetization, but whatever their shape, and however irregular the space disposition of their $M.M.F.$, there will always be some combination of superposed solenoids of various lengths, shapes, and excitations that would form their magnetic equivalents, so that all problems connected with magnets are reducible, ideally at least, to cases involving only the corresponding virtual solenoids.

All these stress distributions are subject to a general law, whose recognition tends to connect and coördinate their various consequences. In any loop, or coil of wire

carrying a current, there is as we have seen, a flux $F = \frac{M}{R}$.

If flexible or extensible, the loop will tend by its motion to augment F , by reducing R . It will not only compress the wire so as to shorten the paths immediately encircling it, but the loop will spread into a circle, the form of minimum reluctance for a given length of conductor, and radially extend that circle until held in check by mechanical

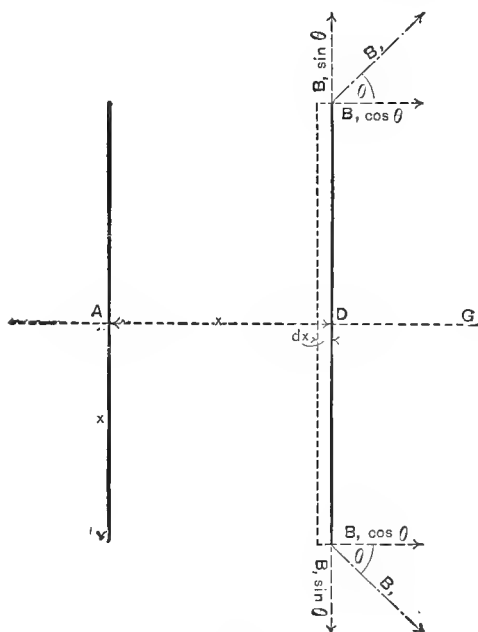


FIG. 31.

forces. Similarly, when the loop is subjected to the flux from an external magnetic circuit, it will tend to enclose a greater total flux, and to move to a position wherein F may become a maximum. Magnetic equilibrium whenever attainable in other positions will be unstable.

As a consequence of the tendency exerted by an active conducting loop or coil, to move from a position of lesser to one of greater flux enclosure, the force urging it any instant along its path to its goal of maximum contents, will

be the space rate of adding new flux at that moment to the flux already encircled. The coil will advance in such a manner that this rate of addition shall be the greatest rendered possible by the flux distribution, under the conditions imposed by the form of the coil and its geometrical freedom. The journey will thus be performed in the least time and by the shortest route that the system will permit.

We may observe this in the simplest case of two equal and coaxial circles of conducting wire maintained at radius distance, as represented in projection by Fig. 31. If unit current flows round circle *A* it will generate a M. M. F. of 4π which will force a total flux of, say F C. G. S. lines through the circuit; or in other words, F will be the surface integral of flux through *A*. A certain share of F (in the case represented by the figure, numerically, 0.8958 r), will pass through circle *D*, and the total flux so intercepted—the surface integral of flux from *A* over *D*'s area—may be called m . This quantity m will be zero when the axial distance between the rings is infinite, will increase with their proximity, and in the ideal case of filamentary circles will numerically equal F when *A* and *D* coincide. Practically, the thickness of the wires forming the circles will limit their approach, so that m can never equal F . It is evident that when *A*'s unit current is altered in strength to c_1 units, the M. M. F. will be altered in like proportion, changing both F and m to $c_1 F$ and $c_1 m$ respectively, but leaving the ratio $\frac{m}{F}$ of the total fluxes through *D* and *A* unchanged.

Round the circumference of circle *D*, the intensity due to the M. M. F. of c_1 will be, by symmetry, uniform in strength and in inclination θ with the axis *A D*, and its value may be denoted by B_1 . As represented in the figure, this inclination will be almost exactly 45° or, $\frac{\pi}{4}$ in this instance, and equal to $1.61 c_1 \div r$ C. G. S. lines per sq. cm. Resolved into components this becomes $B_1 \sin \theta$, in the plane of the circle, radially outwards in all directions from the axis, and $B_1 \cos \theta$ cutting the circumference parallel to the axis. In the case before us these components happen to be each $1.14 c_1 \div r$, the current c being supposed to circulate positively, or counter-clockwise as viewed from *G*. The circle *D* is now filled with flux aggregating to $c_1 m$ lines, terminated at the margin with these component intensities.

Next, suppose that a steady current of c_2 units is estab-

lished in $\mathfrak{D}$, also counter-clockwise, as viewed from G . The stress distribution due to the combined flux systems will be represented by the vector product of $B_1 \sin \theta$ in the plane of $\mathfrak{D}$, upon c_2 round the circumference, urging $\mathfrak{D}$ bodily towards A ; also the corresponding vector product of $B \cos \theta$ on c_2 tending to expand the circle $\mathfrak{D}$ radially, which force its rigidity will hold in check and which may therefore be dismissed from consideration. The summation of $B_1 \sin c_2 r d\theta$ round $\mathfrak{D}$ is

$$P = 2 \pi r B_1 c_2 \sin \theta; \text{ or } P = O B_1 c_2 \sin \theta,$$

where O is the circumference of the circle, and P will be the force of attraction between them. If c_1 and c_2 circulate in opposite directions the same force P will be exerted in repulsion.

As a numerical illustration, applicable to Fig. 31, let the radius r and axial distance x between the circles be 5 cms.; $c_1 = 3$, or 30 amperes; $c_2 = 4$, or 40 amperes. B_1 then, becomes $1.64 \times \frac{3}{5} = 0.966$ at the circumference of $\mathfrak{D}$, and the local components $B_1 \sin \theta$ and $B_1 \cos \theta = 1.14 \times \frac{3}{5} = 0.684$ radially outward, and through the ring. The force of attraction between the circles will then be $P = 0.684 \times 2 \pi \times 5 \times 4 = 85.96$ dynes.

This result can be expressed in terms of m ; for if while c_1 and c_2 are flowing, the circle $\mathfrak{D}$ is given a displacement dx along the axis, as shown, so small that B_1 does not appreciably alter within the range, m will increase by the motion and the total flux from A threading $\mathfrak{D}$ will rise from $c_1 m$ to $c_1 m + 2 \pi r dx B_1 \sin \theta$; for the surface integral of A 's fluxes over $\mathfrak{D}$ in its new position will, by the results of page 4, be equal to the surface integral over $\mathfrak{D}$ in its previous position plus that over the cylinder dx generated by the circumference during the small displacement. This latter must be the intensity normal to the cylindric surface, namely, $B_1 \sin \theta$ into the area of that surface, or $O dx$, making $O B_1 \sin \theta dx$ in this instance numerically

$$1.14 \frac{c_1}{r} \cdot 2 \pi r dx = 2.28 \pi c_1 dx,$$

and this will be the increase in $\mathfrak{D}$'s share of A 's flux for a small excursion dx of $\mathfrak{D}$. If this rate of increase were to continue unaltered for one centimetre of displacement, the increment would amount to $c_1 B_1 \sin \theta O$ total lines, representing an increase in the value of m to the amount of $B_1 \sin \theta O$, and since $P = c_2 B_1 \sin \theta O$ it follows that the attractive force between the circles is the product of

the two current strengths and the rate at which m increases per cm. of advance, as gauged from a small displacement; or in symbols

$$P = c_1 c_2 \frac{dm}{dx}.$$

If P attains the negative sign in this formula, the pull will be exerted in the direction opposite to that of the infinitesimal displacement dx .

The share intercepted by A of flux due to unit current in D , must by symmetry be equal to D 's interception of A 's flux from unit current; that is to say m has a mutually reciprocal value between the circles. Again, if instead of being coaxial, A and D be removed to any distance and set at any inclination θ to each other, m will still be reciprocally equal. For, viewed from the centre of circle A , D will be seen at distance l , say, and under inclination θ . On

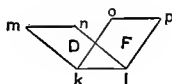


FIG. 32.

shifting the standpoint to the centre of D , A 's distance and aspect must be relatively the same.

Further, we may remove the remaining limitations of equality and form, by proving that m will be mutual between any two circuits under any fixed geometrical conditions whatsoever. For in Fig. 32 let A , D , and F , be three equal squares fixed in space, D and F having one common side. If unit current flows round the contours of these squares, then the flux from A intercepted by F , say A_F must be equal by the above reasoning to the flux from F intercepted by A or F_A . Similarly $A_D = D_A$. Summing these two equalities, the flux in F and in D together, due to A , must be equal to that in A due to D and F coincidentally, or $A_D + F = D_A + F_A$. The currents in the common side kl of D and F oppose and neutralize, so that kl may be removed without influencing this result. When this is done, the flux intercepted by A from unit current round $m k o p l n$,

will be equal to the flux intercepted by $m k o p l n$ from that due to unit excitation in λ . Continuing this process, any loops or series of loops can be constructed, having any sizes or shapes, and it will be evident that they have mutual and reciprocal values of m . This quantity m is called the mutual inductance of the two circuits. The proposition only holds good for non-magnetic media, and the introduction of iron into the system induces more or less deviation from the rule.

For a complex system of conducting loops, such as a pair of coils with many turns, situated in given relative positions, we might proceed to find m by establishing unit current in one coil, and then determining the flux due to this M. M. F. that is intercepted by each turn of the other coil, taken separately, and as though existing alone. Let $m_1, m_2, m_3, \dots m_n$, be the separate interceptions of the n turns in the recipient coils, then the mutual inductance of the two coils will be the sum of these terms, or Σm .

If an active coil made up of any arrangement of turns, be placed in a uniform prime flux due to an external M. M. F., it is clear that no motion of translation can alter m , although a rotatory displacement will in general do so. The force tending to rotate the system round any axis will have a moment

$$t = c_1 c_2 \frac{dm}{d\theta};$$

where $\frac{dm}{d\theta}$ is the rate of increase in the mutual inductance per radian or unit of angular displacement round this axis. The coil, if free, will rotate round that axis which possesses

the greatest values of $\frac{dm}{d\theta}$ and t . If its freedom be restricted to motion round special axes or in certain planes, the coil will still select that axis of rotation left free which has the greatest value t . The moving forces will vanish so soon as m attains a maximum, but frequently, the momentum acquired by the moving system in its rotation to this position will carry it beyond, against opposing forces, and set up an oscillation about the position of ultimate repose. This is the recognized general behavior of active coils and of magnets, their prototypes, when suspended in prime fluxes that are practically uniform.

When the prime flux is not uniform, a translatory motion will, in all but exceptional cases, increase the mutual in-

ductance of the systems, and the moving forces will in general consist of a resultant force accompanied by a couple. The motion of the system will be that translation and rotation consistent with geometrical freedom, that will increase the mutual inductance most rapidly in space and time.

Although the distribution of stress in a magnetic circuit may always be found by summing the vector products of flux and current on each element of conductor, it is some-

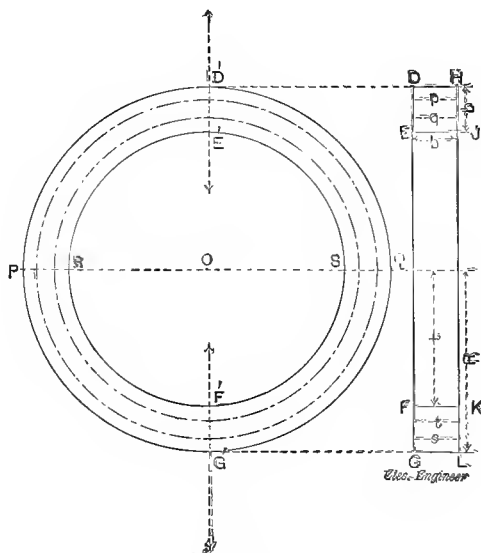


FIG. 33.

times more readily obtained by determining the local values of $\frac{B^2}{8\pi}$ throughout the system, and from these their resultants.

As an example of some practical importance, we may consider a closed hollow ring or circular solenoid with a uniform winding of N turns, a square cross section of A sq. cms. $= b^2$, and generating radii of r and R cms. as represented in plan and diametral section by Fig. 33. By preceding results, we know that when excited by a current of C units, the intensity B within the core at any

radius ρ is $\frac{2 N C}{\rho}$. The flux paths are all circles and

exert a tension of $\frac{B^2}{8\pi}$ dynes per sq. cm. along these circles,

but which tensions are nowhere directly impressed upon the walls of the ring. If the intensity within the ring were uniform, the total tension exerted across any section

PR of the hollow core would be $\frac{A B^2}{8\pi}$, but since the in-

tensity diminishes outwardly, the total tension must be found by subdivision of the area into elements and summing the elementary tensions on each. In this instance the

surface integral of the tension $T = \int_r^R \frac{B^2}{8\pi} dA$, where

$B^2 = \frac{4 N^2 C^2}{\rho^2}$ and dA becomes $b \cdot d\rho$, so that

$$T = \frac{4 N^2 C^2 b}{8\pi} \int_r^R \frac{d\rho}{\rho^2} = \frac{N^2 C^2 b}{2\pi} \left(\frac{1}{r} - \frac{1}{R} \right) \text{ dynes,}$$

and this is the stress exerted across any radial sections such as PR and SQ when the thickness of the conductor wound on the ring need not be considered.

The stress that is directly brought to bear upon the walls of the core consists of the side thrust or outward pressure of

$\frac{B^2}{8\pi}$ dynes per sq. cm. The stress on the plane surfaces

$DEFG$, and $HJKL$ being equal, opposite, and equilibrating through the substance of the walls, may be omitted from consideration, reserving for examination the curved surfaces $RESR$, and $PDQG$. At any point on the former

and outer circumference, B has the value $\frac{2 N C}{R}$ and

the pressure is $\frac{1}{8\pi} \times \frac{4 N^2 C^2}{R^2}$ dynes per sq. cm. radially

outwards. At any point on the inner surface, B is $\frac{2 N C}{r}$

and the pressure is $\frac{1}{8\pi} \times \frac{4 N^2 C^2}{r^2}$ dynes per sq. cm. di-

rected towards the center o . Two symmetrical systems of stress are thus established, one resident at the outer circum-

ference and directed radially outwards, the other resident at the inner circumference and directed radially inwards.

Selecting any diameter, such as $P O Q$, the resultants of the first system directed across this plane are stresses

$\frac{b N^2 C^2}{\pi R}$ dynes along $O D'$, and $O G'$, while the resultants of

the second system are two equal stresses of $\frac{b N^2 C^2}{\pi r}$ dynes

along $E' O$, and $F' O$, respectively. These forces do not equate but leave a balance directed inwards of

$\frac{b N^2 C^2}{\pi} \left(\frac{1}{r} - \frac{1}{R} \right)$ dynes along $E' O$, and $F' O$. This will

be the stress which, if the ring were actually divided by the plane $P O Q$, the two halves would mutually exert upon each other, or the magnetic attraction which they would evince. Its amount would be equally shared between the two sections $P R$, and $S Q$ by symmetry, and each section

would support a stress of $\frac{b N^2 C^2}{2\pi} \left(\frac{1}{r} - \frac{1}{R} \right)$

The stress exerted tangentially through the walls of the solenoid may be ascribed to the resultant side thrust from the flux and is here equal to the total tension exerted by the flux within the interior. This proposition has very general applications, for in any active coil it is evident that equilibrium could not exist across any section of the coil, unless the stress supported by the substance of the walls counterbalanced the total tension of the flux, and the resultant tension is generally more readily determined than the resultant pressures that equate it.

As a numerical illustration of the foregoing case presented in Fig. 33, let $r = 10$, $R = 14$, $b = 4$, $N = 2000$, $C = 2$ (absolute units) or 20 amperes. Then the total force exerted across any diameter of the ring would be

$$\begin{aligned} T &= \frac{b N^2 C^2}{\pi} \left(\frac{1}{r} - \frac{1}{R} \right) \\ &= \frac{4 \times 2000 \times 2000 \times 2 \times 2}{3.1416} \left(\frac{1}{10} - \frac{1}{14} \right) \\ &\approx 581,900 \text{ dynes} = \frac{581,900}{981} = 593.2 \text{ grammes weight.} \end{aligned}$$

If the ring were actually divided by a diametral plane $p q$ the upper half, suspended at its centre E^1 , would support the lower half vertically beneath it by magnetic force under these conditions, if the mass of that lower half did not exceed 593 grammes, each section, $p r$ and $s q$ exerting a total stress of 296.5 grammes weight.

If the difference of intensity over the cross-section of the ring were negligible, the tension exerted across any

section would reduce to $A \cdot \frac{B^2}{8\pi}$ simply, or $\frac{A}{8\pi} \cdot \frac{4 N^2 C^2}{r^2}$

and the total stress acting across any diameter of the ring

would be double this or $T = \frac{A N^2 C^2}{\pi r^2}$. Thus in the

last case, with a mean radius of 12 cms.,

$$T = \frac{16 \times 2000 \times 2000 \times 2 \times 2}{3.1416 \times 12 \times 12}$$

$$= 565,800 \text{ dynes} = 576.8 \text{ grammes weight.}$$

The tension across each section being 285.4 grammes weight and by the adoption of the approximate formula, the stresses would have been here underestimated by about three per cent.

The formulæ indicate that the tensions exerted across a section of any closed circular solenoid without iron, are proportional to $(N C)^2$ or the square of the current turns. This is a direct consequence of the facts that B is proportional to the current turns, and the tension in its turn is proportional to B^2 .

We may next consider the distribution of stress within a double solenoid or pair of concentric rings, one of which is enclosed within the other. Such a system is related to the practical case of a solenoid with more than one layer in its winding. In Fig. 33, the dotted lines through $p q s$ and t represent the walls of an internal concentric ring of radii r_1 and R_1 , current C_1 and N_1 turns. All the walls of the double solenoid may be supposed rigid, but of negligible thickness.

Exciting the outer coil only, the intensity at any point within it of radius ρ will be $B_o = \frac{2 N C}{\rho}$. Exciting the inner coil only, the intensity will be $B_1 = \frac{2 N_1 C_1}{\rho}$ at any

point within its walls, but zero beyond them. Exciting both coils simultaneously, the intensity at any point in the internal ring will be that due to both coils, or

$$B_0 + B_1 = \frac{2}{\rho} (N C + N_1 C_1.)$$

For a point within the outer ring but without the inner,

$$B_0 = \frac{2 N C}{\rho} \text{ remains undisturbed.}$$

The stress per sq. cm. under these conditions will be $\frac{(B_0 + B_1)^2}{8\pi}$ within the inner ring, and $\frac{B_0^2}{8\pi}$ within the outer only. Across any section such as s q, the surface integral of tension over the inner ring will be

$$t_1 = \frac{b (N^2 C^2 + 2 N N_1 C C_1 + N_1^2 C_1^2)}{2\pi} \left(\frac{1}{r_1} - \frac{1}{R_1} \right)$$

Similarly between radii r and r_1 of the outer ring the total tension will be $t_2 = \frac{b N^2 C^2}{2\pi} \left(\frac{1}{r} - \frac{1}{r_1} \right)$ and again

between radii R_1 and R , $t_3 = \frac{b N^2 C^2}{2\pi} \left(\frac{1}{R_1} - \frac{1}{R} \right)$. The

total tension across the entire section must be $T = t_1 + t_2 + t_3$.

It might, at first sight, be supposed that since the total tension within the inner ring is t_1 , its walls must sustain a stress of this amount. If, however, the side thrust pressures due to the intensities adjacent to the walls, be summed and compounded, it will be found that the stress on those of the inner ring will only be

$$\frac{b (N_1^2 C_1^2 + 2 N N_1 C C_1)}{2\pi} \left(\frac{1}{r_1} - \frac{1}{R_1} \right),$$

while the balance of t_1 , or $\frac{b N^2 C^2}{2\pi} \left(\frac{1}{r_1} - \frac{1}{R_1} \right)$ is sup-

ported by the stress from the external intensity B_0 , and is carried over to the outer coil. The stress supported by the walls of the outer ring is thus increased to

$$\frac{b N^2 C^2}{2\pi} \left\{ \left(\frac{1}{r} - \frac{1}{r_1} \right) + \left(\frac{1}{r_1} - \frac{1}{R_1} \right) + \left(\frac{1}{R_1} - \frac{1}{R} \right) \right\}$$

across any section, or simply $\frac{b N^2 C^2}{2 \pi} \left(\frac{1}{r} - \frac{1}{R} \right)$; and

this would be the stress on those walls if the inner ring were absent or remained unexcited.

In a compound circular solenoid, therefore, the total stress sustained by all the walls together is equal to the surface integral of tension over the whole area to the limits enclosed by the outermost, but its distribution among the rings is such, that the inside coils take less than the integral tensions over their own sections, leaving the balance to the walls outside. This tends to secure a more uniform distribution of the total tension on the walls. The outside layer always carries the stress that it would support if all the inner rings were removed or left unexcited.

The introduction of iron into the magnetic circuit produces no change in stress principle, but usually a great change in its degree. If the magnetized iron be regarded as the equivalent of a certain solenoid, or combination of solenoids, the magnetic stresses that would be developed in that equivalent solenoid, are developed in the iron. While, however, we have seen that equilibrium of stress exists throughout the hollow core of a simple coil or solenoid, and that stress is only manifested in the walls, or turns of active conductor, the stress in iron is developed throughout its substance. A uniformly magnetized bar of iron when divided across by any plane exhibits attractive force between the severed portions all over their opposed areas, just as its equivalent solenoid would do, if the insulated wire was spirally wound in a groove on a core of copper or silver, so rigidly that the stresses exerted by the wire became imparted to the encircled substance. Apart from the artificiality of the hypothesis by which an equivalent solenoid is substituted for a magnet, this distribution of the stress through the substance of the iron must follow from the molecular nature of magnetization demanding in strictness a correspondingly large number of molecularly small solenoids to adequately represent the theory, whereas, for practical magnetic purposes a single solenoid representing the entire mass might give sufficient approximation.

The stress distributions in magnetic circuits containing iron may next be examined, commencing with the simplest case. Let us take a closed circular solenoid of which the section S is so small relatively to the generating radii, that the intensity within it is practically uniform. This inten-

sity will be $B = \frac{2 N c}{r}$. The total stress exerted along

the flux paths will be $\frac{B^2}{8 \pi} = \frac{N^2 c^2}{2 \pi r^2}$ and the total amount

of this over the section S will be $\frac{S N^2 c^2}{2 \pi r^2}$. Suppose $r =$

15 cms. $S = 2$ sq. cms., $N = 500$ turns, and $c = 0.5 = 5.0$

amperes; we have $B = \frac{2 \times 500 \times 0.5}{15} = 33.3$ c. g. s. lines

per sq. cm. so that the total tension will be $\frac{2 \times 500^2 \times 0.5^2}{2 \times 3.14 \times 15}$

$= 88.4$ dynes, and this will be the stress exerted along the walls of the solenoid, across any radial section.

If now a soft iron core be inserted so as to fill the solenoid, the prime intensity of 33.3 will excite a total resultant intensity in the iron of 33.3μ or perhaps 16,650 c. g. s. lines per sq. cm., since μ would be about 500 for this

excitation. The stress for this intensity would be $\frac{(16,650)^2}{8 \pi}$

or approximately 11,000,000 dynes per sq. cm., and the total stress, 22,000,000 dynes, will be supported by the iron core and the solenoid together. Since we may regard the core as magnetically equivalent to an independent internal solenoid of definite excitation, we know by the results reached in the preceding chapter, that the stress on the walls of the outer solenoid will be, so far as mechanical freedom will permit, equal to that sustained before the insertion of the inner solenoid, in this case the iron core. The coil in this case will therefore retain the stress of 88.4 dynes, and the remainder, or 21,999,911.6 dynes, will be supported by the iron core, assuming that coil and core are mechanically independent or frictionless. It is evident that in nearly all cases that can practically occur, the share of stress supported by the solenoid is relatively so small when it encloses iron, that the core may be regarded as exerting the entire stress.

If the ring were divided into halves by a plane through the axis of revolution, each section would support 22 megadynes, and the total attraction between the two halves would be 44 megadynes, or 44.85 kilogrammes weight.

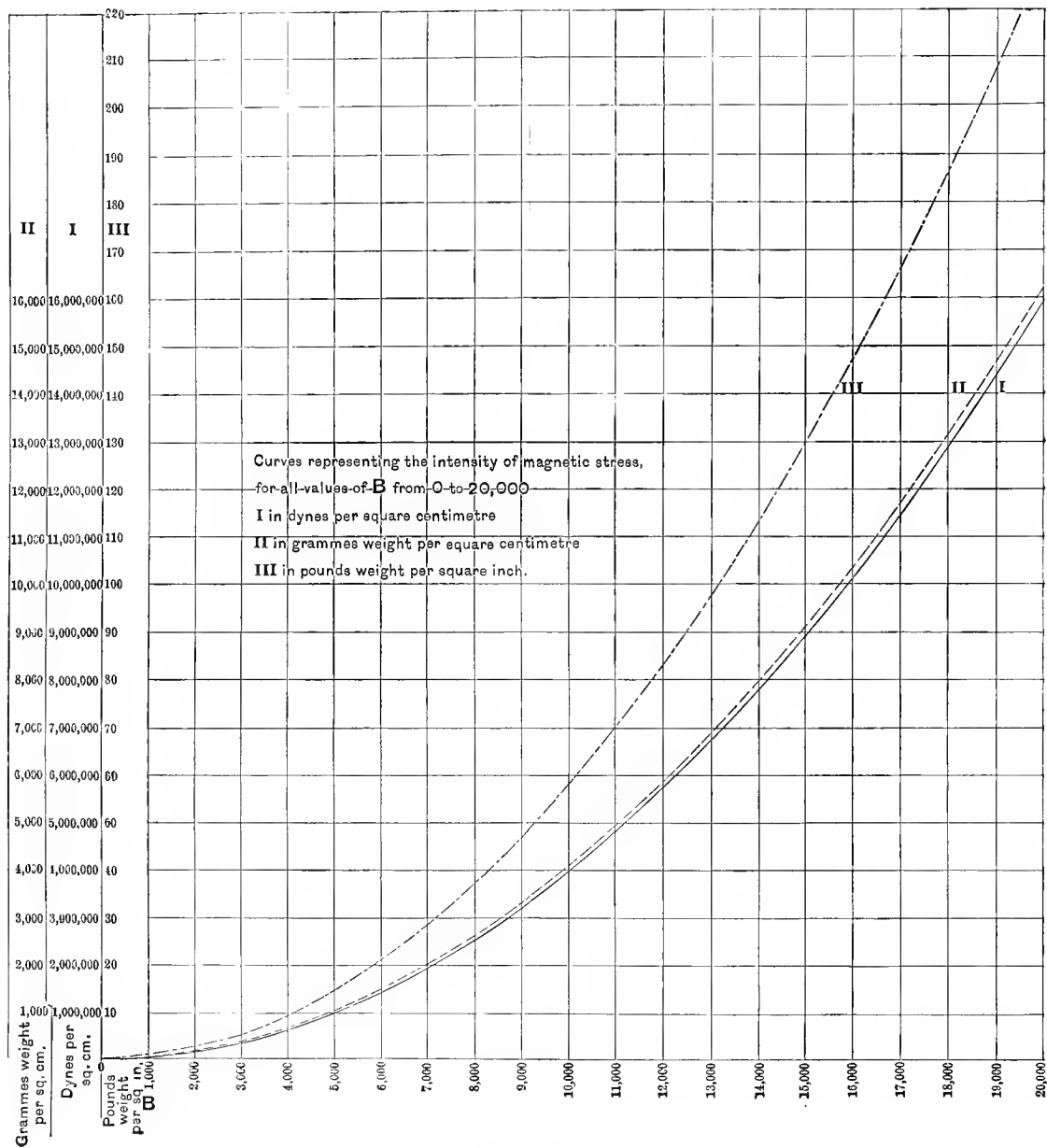


FIG. 34.

The introduction of the iron core has increased the power 250,000 times.

Fig. 34, gives the curves representing the value of $\frac{B^2}{8\pi}$

in dynes per sq. cm., grammes weight per sq. cm., and pounds weight per sq. inch, enabling the tension corresponding to any flux density to be determined by inspection. Thus, if two smooth plane surfaces of iron are brought into contact, and a uniform intensity of 14,000 passes perpendicularly across the plane of junction, the attraction between those surfaces will be 7.8 megadynes, or 8 kilogrammes per sq. cm., or 113 lbs. per sq. inch of contact surface. If the plates are not in contact, but lie parallel to and facing each other, their attraction will still be the same if the intensity remains unaltered in value or distribution. In practice, of course, this is not generally the case, and the intervention of an air-space between the plates will cause the flux to diminish and to deviate from the perpendicular in its passage.

Suppose that the air gap between two parallel and similar opposed pole faces is d cms. across, the intensity at, and near, the polar edges, will be curved and variable from point to point, but at any point in the air gap between the poles not less than $\frac{d}{2}$ cms. from the polar edge the inten-

sity B_d say, will be practically uniform, and perpendicular across, so that the stress, or attraction between the poles will be $\frac{B_d^2}{8\pi}$ dynes per sq. cm. of surface, and less than this near the edges.

The next case in order of complexity, is that of a closed circular solenoid in which, owing to the area of cross-section, the intensity may not be regarded as uniform. The prime intensity varies from $\frac{2 Nc}{r}$ at the inner wall to $\frac{2 Nc}{R}$

at the outer. With an iron core, the value of μ will generally differ at these limits, and the resultant intensities will tend to approach more closely than their excitations. The value of B will differ at different points in a section, not only on account of difference in the radius, but also owing to the variation in μ due to the different excitations thus established. We can no longer obtain the total stress

from the product of $\frac{B^2}{8\pi}$ and the cross sectional area, but

some process of average or summation must be resorted to, according to the degree of accuracy desired.

As a numerical instance, suppose $R=15$, and $r=10$, to be the outer and inner generating radii of the soft iron core whose breadth is $h=5$, and section square, $N=600$, and $c=3$ amperes, $=0.3$. Then the prime intensity at the inner

wall is $\frac{2 \times 500 \times 0.3}{10} = 36$, and at the outer wall $\frac{2 \times 600 \times 0.3}{15}$

$= 24$. For $H = 36$, μ might be 400 in soft iron, and for $H = 24$, 500, so that the resultant intensity would range from $24 \times 500 = 12,000$ at the outside to $36 \times 400 = 14,400$ at the inside of the core, the corresponding stresses being 5.85 and 8.4 kilogrammes per sq. cm. respectively. As a first approximation to the total stress, we might take the mean value of these extreme limits or 7.125, and the area of cross section, 25 sq. cms., making a product of 178.1 kilogrammes weight. A closer result would be obtained by determining a series of prime intensities for intermediate radii, multiplying each by its corresponding μ , and then

taking the average of the $\frac{B^2}{8\pi}$ values. Or an empirical re-

lation could be assigned by trial between μ and B so that the total stress could be determined by integral calculus. The limit of accuracy would be ultimately determined by the precision with which the scale of μ could be experimentally determined.

Now suppose the polar surfaces reduced to 1.5 sq. cms. each. With the same total flux through the armature, the density at the contact surfaces will be 13,333, representing a stress of 7.3 kilogrammes per sq. cm., or a total stress at each pole of 10.95, and for both poles combined 21.9, an increase of 5.68 above the preceding case. We have here assumed that the total flux of 20,000 lines passed through the armature was the same in each case, but in reality there would be some reduction in the flux as the polar surface diminished, and this reduction would become more marked as the polar area reduced, so that finally when the flux density in the soft iron at the polar areas approached 18,000 c. g. s. lines per sq. cm., or in other words, as the iron at those areas approached saturation, the advantage gained by

increasing B would be just offset by the reduction in the total flux that would pass through the surfaces. Beyond this point of maximum effect, still further reduction of the polar contact areas would result in a lessened total attractive force. This maximum would depend upon all the conditions of the magnetic circuit, and its determination may be left for later consideration.

These principles also explain the advantage that is secured for permanent magnets of hard steel by softening their poles when these latter are of due proportions. Hard steel is necessary for their construction in order that the counter-magnetic flux from their poles may not demagnetize them, but at the polar surfaces, it is desirable to have B^2 , and therefore B , as great as possible for a large tractive power, and soft steel will more readily permit a high intensity to be obtained.

Similarly they indicate the necessity for employing a permanent magnet in a receiving Bell telephone in place of a soft iron core. Assuming that the diaphragm of ferrotype plate is so close to the pole, that the flux density may be regarded as uniform and everywhere perpendicular to the surfaces, the maximum attractive force due to a soft iron core without a permanent magnet, and under the influence

of an alternating current in the coil, will be $\frac{A B_c^2}{8 \pi}$, where A is the effective polar area, and B_c is the maximum intensity that the alternating current can effect. The insertion of the bar magnet produces a steady normal pull on the diaphragm

of $\frac{A B_m^2}{8 \pi}$ dynes, where B_m is the permanent magnetic intensity. If this is not nearly sufficient to saturate the iron at the polar surfaces, we may consider that B_c may be super-

posed upon B_m when the alternating current passes through the coil. The stress exerted on the disc then alternates

between $\frac{A}{8 \pi} (B_m + B_c)^2$ and $\frac{A}{8 \pi} (B_m - B_c)^2$, a total range

of $\frac{4 A B_m B_c}{8 \pi}$ which is generally much greater than the

original value $\frac{A B_c^2}{8 \pi}$ and the electromagnetic effect of the coil becomes correspondingly augmented in the ratio of $4 B_m$ to B_c .

Experiment shows that the friction to sliding between two smooth or nearly smooth surfaces of iron under mag-

netic stress, is not sensibly different from that accompanying the same stress exerted mechanically, so that if f is the coefficient of friction (sensibly 0.2) between ungreased smooth surfaces of iron, the stress that it will be necessary to apply between them parallel to the plane of contact will be $\frac{f}{8\pi} \int B^2 ds$, or about one-fifth of the total tractive force.

The subject of tractive power of magnets has been extensively developed in Professor S. P. Thompson's "Electromagnet."

The fact that the intensity of magnetic stress varies as the square of the flux density, and that the total stress is the ultimate sum of all the intensities, into the area over which they are exerted, as expressed by the equation

$$P = \frac{1}{8\pi} \int B^2 ds,$$

is the clue to all the most important phenomena of attraction between magnets. It explains, for example, the seeming paradox that an electromagnet will often support a heavier weight from its armature after the contact surface between poles and armature is reduced. For if the flux through the armature is say 20,000 lines, and the polar surfaces two sq. cms. each, the density will be 10,000 per sq. cm. corresponding, by the curve of Fig. 34, to a stress of 4.056 kilogrammes per sq. cm. and the total pull will be 8.11. Since this stress will be exerted at each pole, the keeper should support at a point midway between them a total weight of 16.22 kilogrammes minus its own weight.

We may next consider the stress exerted upon magnetizable particles situated in a magnetic field. This case is practically represented in the moving force of iron filings toward magnets. For the sake of simplicity we may first suppose that the attracted particles are all of spherical form.

When an iron sphere of radius r , and permeability μ is placed in a uniform prime flux of density B_p , we may imagine that every elementary rod, or line of particles in the sphere, parallel to B_p , becomes magnetized, and establishes its own local magnetic circuit. Part of the induced flux in completing each rod's circuit, permeates the substance of all the other rods, more or less obliquely, but in a direction opposed to the prime intensity B_p . Every rod is thus acted upon by B_p in one direction, and by the return or counterflux from the rest of the sphere in the

opposite way, and the induced M. M. F. in the iron is locally $\mu-1$ times the resultant B_r of these opposing fluxes. The ellipsoid is one of the few geometrical forms for which the resultant has been determined, and under this classification the sphere makes its appearance as a particular case. It can be shown that within the substance of a magnetizable sphere the counterflux is uniform and just one-third of the induced intensity B_i , so that the resultant prime intensity B_r throughout the sphere is

$$B_r = B_p - \frac{B_i}{3}; \text{ and since } B_i = (\mu-1) B_r,$$

we must have

$$B_r = \frac{3 B_p}{\mu + 2} \text{ and}$$

$$B_i = (\mu-1) \frac{3 B_p}{\mu + 2}.$$

The total intensity within the sphere B_s being the sum of the resultant and induced intensities, represented by these last two equations, will be

$$B_s = \frac{3 \mu B_p}{\mu + 2}.$$

Thus with a sphere of Norway iron, these component internal intensities would assume values dependent upon the prime flux density in a manner outlined by the following table. The values of μ are supposed to have been determined either directly, or by correspondence with known samples of iron.

Prime Intensity B_p .	Resultant of prime and counterflux in- tensities B_r .	μ .	Intensity induced in sphere by its M. M. F. B_i .	Total Intensity in sphere ($B_r + B_i$). B_s .
1	$\frac{3}{152} = 0.0197$	150	$\frac{1 \times 3 \times 149}{152} = 2.941$	$\frac{1 \times 3 \times 150}{152} = 2.9607$
2	$\frac{3 \times 2}{162} = 0.0370$	160	$\frac{2 \times 3 \times 159}{162} = 5.890$	$\frac{2 \times 3 \times 160}{162} = 5.927$
10	$\frac{3 \times 10}{222} = 0.1351$	220	$\frac{10 \times 3 \times 219}{222} = 29.60$	$\frac{10 \times 3 \times 220}{222} = 29.7351$
100	$\frac{3 \times 100}{602} = 0.498$	600	$\frac{100 \times 3 \times 597}{602} = 298.5$	$\frac{100 \times 3 \times 600}{602} = 298.998$
1,000	$\frac{3 \times 1,000}{3,002} = 0.999$	3,000	$\frac{1,000 \times 3 \times 2,999}{3,002} = 2,997.$	$\frac{1,000 \times 3 \times 3,000}{3,002} = 2,998.$
5,000	$\frac{3 \times 5,000}{502} = 29.88$	500	$\frac{5,000 \times 3 \times 499}{502} = 14,910.$	$\frac{5,000 \times 3 \times 500}{502} = 14,940.$

It is evident that the counterflux nearly neutralizes the prime intensity within the sphere for all ordinary conditions, while the induced, and also the total intensity are almost three times the prime. Immediately outside the sphere, the distribution of flux is of course no longer uniform, since the prime flux is here compounded with the return flux from the sphere which completes its local circuit by curving paths, but the equipotential curves for this local circuit correspond in outlines to those represented in Fig. 5, page 13. The table and formulæ also show that the value of μ may vary through wide limits without much alteration in the total intensity, so that if we take spheres of iron, nickel, cobalt, or even ferroso-ferric oxide, and place them in a uniform magnetic field, the intensity within their substance will be uniform, and usually not far from three times the prime.

Having then determined the values of the uniform prime intensity, and the corresponding intensity within the sphere from the assumed range of μ , we have next to ascertain the stress which this distribution will exert upon the sphere as a rigid body. For this purpose we may replace the iron sphere by its equivalent active coil, and determine the stresses set up on that coil by the foregoing rules. The coil must be of negligible thickness, and be wound upon a sphere of wood or other non-magnetizable material, having the same dimensions as the iron sphere it replaces. Within the wooden sphere we have to produce

a uniform total intensity of $\frac{3 B_p \mu}{\mu + 2}$, and since B_p is the prime intensity existing within the coil before it is excited, the uniform flux density that the coil must create within it, and superpose, is

$$\frac{3 B_p \mu}{\mu + 2} - B_p = 2 B_p \left(\frac{\mu - 1}{\mu + 2} \right),$$

and this practically becomes $2 B_p$ when μ has a large value.

We have already seen in Chap. III, that a winding of C current turns per axial centimetre disposed on the surface of a non-magnetizable sphere, produces everywhere within it the uniform flux density of $\frac{8 \pi C}{3}$, and consequently the current turns on this wooden sphere must be

$$C = \frac{3}{8 \pi} \times 2 B_p \left(\frac{\mu - 1}{\mu + 2} \right) = \frac{3 B_p}{4 \pi} \left(\frac{\mu - 1}{\mu + 2} \right)$$

per centimetre slice perpendicular to B_p . All that now remains is to find the magnetic stress on the coil with this excitation.

Taking any thin slice in the winding and considering it separately (Fig. 35), we have an active ring at right angles to a uniform prime flux B_p . There will be a stress radially outwards all round the ring, which its rigidity will hold in check, but there will be no tendency to motion along the line of B_p in either direction, and consequently the entire coil, or the iron sphere, its equivalent, will be in equilibrium so far as regards resultant magnetic stresses. It follows, therefore, that a homogeneous sphere of iron or magnetic

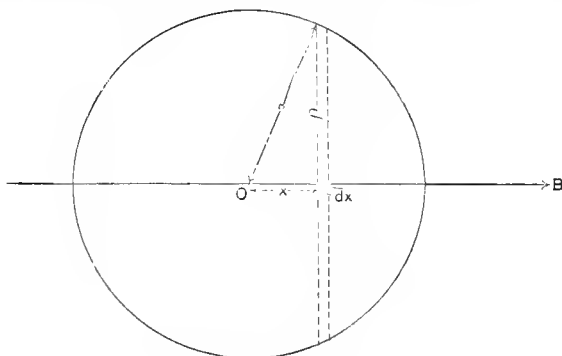


FIG. 35.

material will remain at rest in a uniform magnetic field of any intensity, just in the same way that a compass needle freely suspended in the earth's flux (assumed to be uniform) exhibits directive tendencies, but no inclination to move bodily towards either geographical pole.

Suppose, however, that the prime intensity B_p in which the iron sphere rests, is no longer uniform, but increases steadily by b c. g. s. lines for every centimetre of distance along the line of B_p , while the radius r of the sphere is so small relatively to b that the variation of the intensity existing within its interior may be neglected. Then if the equivalent active coil were a single plane loop perpendicular to B_p of radius ρ carrying a current C , we know that the resultant stress on this loop would be $b \pi \rho^2 C$ dynes along the line of B_p , for the total increase in the flux enclosed by the loop, per centimetre of its excursion along B_p would be

b times the area, and the resultant stress is this rate of flux absorption multiplied by the current in the loop.

Since the coil in this case is a spherical shell we have to divide it into elementary loops and sum their respective resultant stresses. In Fig. 35, at any distance x from the centre of the sphere measured along the line OB , the radius of any elementary slice of thickness dx is

$$\rho = \sqrt{r^2 - x^2}$$

and the equivalent current round this slice is

$$C \, dx = \frac{3 \, B_p}{4 \, \pi} \left(\frac{\mu - 1}{\mu + 2} \right) dx.$$

The elementary pull on the slice is then $dP = b \, C \, \pi \, \rho^2 \, dx = b \, C \, \pi \, (r^2 - x^2) \, dx$ and the total pull on the sphere will be the integral of this expression between the limits $x = \pm r$,

$$\text{or } P = b \, C \, \frac{4 \, \pi \, r^3}{3} = \frac{3 \, b \, B_p}{4 \, \pi} \left(\frac{\mu - 1}{\mu + 2} \right) \frac{4 \, \pi}{3} r^3$$

$$\therefore P = b \, B_p \, r^3 \left(\frac{\mu - 1}{\mu + 2} \right).$$

Also if the variation of prime intensity be no longer simply b per cm., its local rate of increase or gradient will be expressed in all cases by

$$\frac{d \, B_p}{dx}, \text{ and } P = r^3 \, B_p \frac{d \, B_p}{dx} \left(\frac{\mu - 1}{\mu + 2} \right)$$

which, since μ is generally large, becomes practically

$$P = r^3 \, B_p \frac{d \, B_p}{dx} \text{ dynes.}$$

These last two equations embody all the principal facts concerning the apparent attraction between magnets and spherical magnetizable particles. This attraction is nearly the same in the same magnetic flux, and for spheres of the same diameter, whether the substance of the sphere be iron, steel, nickel, cobalt, or Fe_3O_4 , since in all these substances, the factor $\left(\frac{\mu - 1}{\mu + 2} \right)$ will not usually be far below unity.

It is evident that the pull, varying with the cube of the radius, is proportional to the volume of the particle, so that if a given mass of iron be divided into spherical portions, and the distribution of prime flux round all of these is similar, the total pull on the whole mass will be the same, no matter how small the individual spherical diameters may become. In practice, however, the crowding

of a number of spheres into close proximity will influence in some degree the flux distribution. For while the flux density might be sensibly uniform for example, when the spheres remain scattered apart, when closely packed the prime intensity round any one sphere will contain the additional return local flux external to neighboring spheres.

The formulæ also show that if the resultant attractive force is to be made as great as possible $\left(B \frac{dB}{dx} \right)$ must be a

maximum, and it is useless to provide a powerful flux unless its gradient is also considerable, a consideration that merits a more detailed examination.

Suppose a small wooden cylinder, drum-shaped, say 1 cm. long, and also 1 cm. in diameter, whose two faces are denoted respectively by the letters $\mathfrak{E}$ and $\mathfrak{X}$, to be placed in a magnetic flux, and so fixed, that $\mathfrak{E}$ is perpendicular to the direction of B , so that the flux enters normally on that face. Then, because there is no absorption or generation of flux within the cylinder, this quantity entering, namely, 0.7854 B c. g. s. lines, must emerge from the drum, either through the sides, or through $\mathfrak{X}$. If all emerge through $\mathfrak{X}$, the flux is uniform within the limits of the cylinder, whose sides act as a channel or tube of flow. If, however, some flux emerges through the sides, the density of the flux finding exit at $\mathfrak{X}$ will be diminished, since the flux is dispersing. If on the other hand the flux is converging, it will enter through the sides and $\mathfrak{E}$, leaving entirely through $\mathfrak{X}$. Suppose in this case that the density at $\mathfrak{E}$ is 1,000 c. g. s. lines, so that 785.4 lines pass through $\mathfrak{E}$; then if 200 more lines come in through the sides, 985.4 will leave through the face $\mathfrak{X}$, and the rate of increasing density along the direction of B will be here 200 lines per cm. or

$$\frac{dB}{dx} = 200. \quad \text{The attraction therefore of such a field upon}$$

particles of magnetizable material placed therein depends upon the local number of lines that would enter or leave the sides of an imaginary small drum set with its axis along the local direction of B , and the direction of motion would always be towards the denser flux. In other words, the prime flux must be either convergent or divergent, if resultant magnetic stresses are to be exerted upon the particles.

The essential accompaniment of such convergence or divergence is a concavity or convexity in the equipotential

surfaces controlling the flux. In fact, if we take an equipotential surface of any shape, and cut out in it, Fig. 36, a little spherical rectangle of sides ds_1 and ds_2 , the flux traversing the rectangle will be $B \cdot ds_1 \cdot ds_2$. Also if r_1 and r_2 be the radii of curvature of the sides ds_1 and ds_2 , respectively,—the radii of circles that would locally coincide with these sides,—measured along the direction of B

positively, it will follow that $\frac{dB}{dx} = B \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$; or, the

rate of change in B is its product with the sum of the local curvatures in any two perpendicular planes. Thus, when the equipotential surface is plane, r_1 and r_2 are both infinite,

and $\frac{dB}{dx} = 0$, or the flux, as we know, is uniform

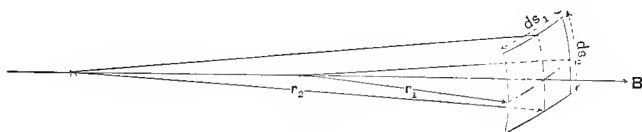


FIG. 36.

Since by this last result the formula for the pull of a prime flux upon a magnetizable sphere may be written

$P = r^3 B^2 \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$, it follows that for a simple mag-

netic circuit—a coil without iron core—the attraction on particles in the circuit will vary at any given point as the square of B , and consequently as the square of the exciting current, for not the shape of the equipotential surfaces, but only their numerical values, will alter with the exciting current, and the sum of the curvatures at any point will remain unchanged. In practice this is nearly true for ferric magnetic circuits with large air gaps such as bar electromagnets, but is of course often far from being true with ferric circuits nearly closed, for near the saturating point of the iron core the changes in the form of equipotential surfaces and of their gradient consequent upon changes of excitation may vary considerably.

In the case of an ordinary permanent or electromagnet, the equipotential surfaces crowd together as the poles are approached, their curvatures at the same time increasing,

so that both B and $\frac{dB}{dx}$ are usually a maximum at the poles, but should the core of the electromagnet be cylindrical, so that its poles are comparatively wide flat surfaces, the equipotential surfaces close to the poles are almost planes, while at the edges they bend much more sharply. The result will be that while there may not be much difference between the values of B (or the distance between successive equipotential surfaces), at emergence from the pole, in the centre, or at the edge, there will be a considerable difference between the rates of flux divergence at these positions, as indicated by the local curvatures, and the product of

$B \frac{dB}{dx}$ will be much greater at the edge. In fact it will

sometimes happen in practice that a powerful electromagnet with broad flat polar faces when plunged into iron filings will become densely packed with these particles round the polar edge, while the centre is left almost entirely denuded. In this or similar circumstances, lies perhaps the foundation for the supposition sometimes entertained, that magnetism principally resides in, or acts at, points. However valuable the curves assumed by scattered iron filings may be in delineating the distribution of flux, the undue prominence that they give to polar points and edges has to be borne in mind. The particles really indicate the

distribution of $B \frac{dB}{dx}$, and not B simply.

Other things being equal, the best form of pole piece for an electromagnet to possess, in order to attract iron particles, is conical. If on the contrary it is required to sustain the largest mass of particles, the flat pole disc is generally preferable, for while the centre will be almost inactive, the long circular edge may more than compensate.

If the flux distribution surrounding a magnet can be determined experimentally, with a field-explorer or otherwise, the attractive force at any point in the field can be computed. Fig. 37 represents the results of a particular case. Curve No. 1 connects successive observations indicated by circles, of the prime intensity along the axis of a large bar electromagnet (excited with constant current), as determined by a field searcher and ballistic galvanometer at each successive centimetre of distance. The intensity B_p descends from about 900 lines at one centimetre from the pole, to 70, at 16 cms. distance. The

local inclination of this curve with the axis of abscissæ as determined by its tangents, gives a series of values for

$\frac{dB}{dx}$ that are represented by curve II. The product at any

distance from the pole of the two ordinates B and $\frac{dB}{dx}$

gives the locus followed by curve III to the left hand scale, and this represents the calculated resultant pull in dynes upon a sphere of iron 1 cm. in radius when μ is large. The pull at various distances along this path was measured in the cases of two iron spheres, one 0.665 cm., the other 1.904 cms. in diameter. The series of observed pulls in dynes divided by the cube of the radius of each sphere, in order to reduce the results to an equivalent sphere of unit radius, are shown by curves IV and V. The agreement between these and curve III is fairly good, and the deviation is perhaps no more than the size of the spheres, or their residual magnetism, might jointly account for in combination with errors of observation.

We have hitherto assumed that the attracted particles are spheres, and of small diameter compared with the flux gradient $\frac{dB}{dx}$. If the particles are not spherical, they will

tend to set lengthwise to the flux, and their local induced counterflux will exercise upon themselves less demagnetizing influence. The equivalent solenoids are therefore stronger, and the resultant stresses greater. In fact, of all forms that particles of given volume free to follow directive tendencies can assume, the spherical yields the least displacement stress. Also, as the particles are elongated, the influence of their permeability becomes more noticeable, and when the length is many diameters, the strength of the equivalent solenoid, and the resultant pull in a given flux increase directly with μ . Such rods, of soft iron, cast iron, steel, or nickel, might, under similar conditions, be expected to differ considerably in attractive force.

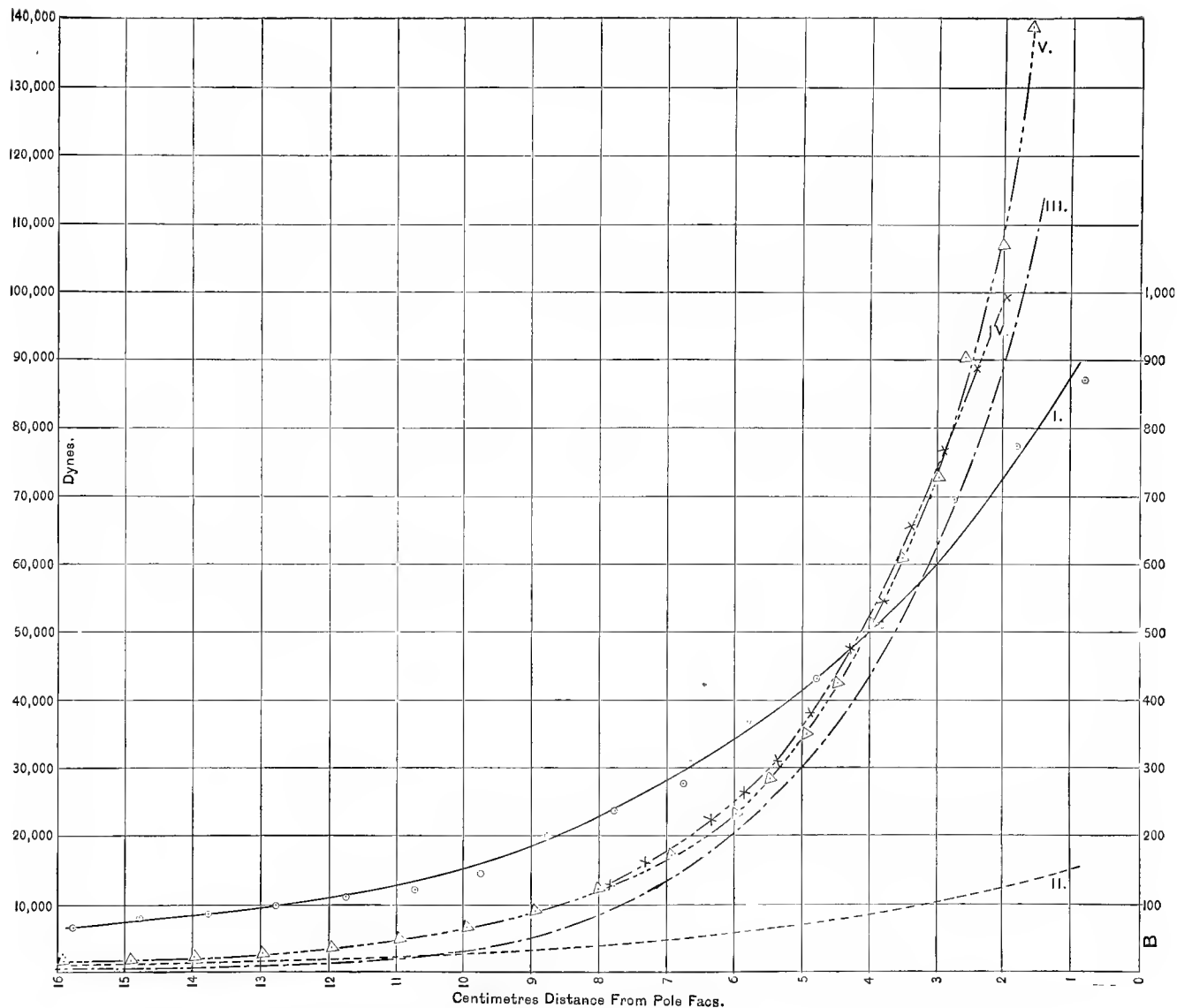


FIG. 37.—CURVES INDICATING THE RESULTANT DISPLACEMENT STRESSES ON IRON SPHERES PLACED IN MAGNETIC FLUX.

Ordinates of right hand scale, flux density (prime) along central line perpendicular to pole face; ordinates of left hand scale, dynes of resultant force; abscissæ, distance from pole face in centimetres. Curve I connects circles indicating observations of flux density by field exploring coil.—Curve II gives the rate of increase of curve I, or its gradient at each point dB/ds (dotted line).—Curve III (broken line) gives at each point the product of the ordinates in curves I and II or $B \times dB/ds$ to left hand scale in dynes on sphere 1 cm. in radius.—Curve IV connects crosses indicating observations of pull $/r^3$ on iron sphere 0.665 cm. in diameter.—Curve V connects triangles indicating observations of pull $/r^3$ on iron sphere 1.905 cm. in diameter.—Under theoretically assumed conditions, curves IV and V should coincide with III.

CHAPTER VI.

Electro-Magnetic Energy.

MAGNETIC flux is a store of energy. It can only be brought into existence by work, and this work, embodied in the flux during its uniform continuance, is again released as the flux wanes and disappears.

If we first consider materials that are practically non-magnetic, so as to temporarily defer the introduction of iron into the subject, then the energy residing in the flux at any point where the intensity is B , amounts to $\frac{B^2}{8\pi}$ ergs per cubic centimetre. A flux density of B , therefore involves a pull along the direction of B , amounting to $\frac{B^2}{8\pi}$ dynes per sq. cm., a thrust all round, of $\frac{B^2}{8\pi}$ dynes perpendicularly outwards per sq. cm. of area, and a store of energy per unit volume to the extent of $\frac{B^2}{8\pi}$ ergs.

Thus the earth's total flux density in the open country near New York being about 0.6 c. g. s. line per sq. cm., the flux energy residing in earth, air, or other non-magnetic substance traversed by this flux, will be $\frac{0.6 \times 0.6}{8 \times 3.1416}$ = 0.0143 ergs. per c. c. and a cubic kilometre of this space will contain 1.43×10^{13} ergs. or 1.43 megajoules of energy in magnetic flux.¹

This equation $W = \frac{B^2}{8\pi}$ ergs. per c. c. is the fundamental relation of magnetic energy.

But there are other expressions for the energy in a magnetic circuit that are often independently useful, and merit consideration.

Apart from permanent magnets, and iron, cobalt, or nick-

1. 1 Joule = 10 millions of ergs = 0.737 foot pound (at latitude of Greenwich).

el, which we still reserve for later examination, magnetic flux is only sustained in general by electricity in motion, that is to say, in particular and in practice, by electric currents;

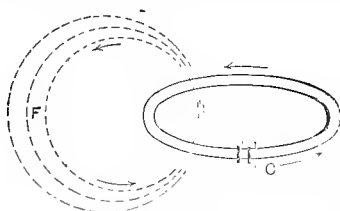


FIG. 38.—DIAGRAMMATIC REPRESENTATION OF SIMPLE MAGNETIC CIRCUIT LINKED WITH ONE TURN OF ACTIVE CONDUCTOR.

c = units of current. F = encircled flux. Energy, $W = \frac{c F}{2}$ ergs.

and its magnetic energy may be expressed in terms of the current strength and the flux enclosed thereby. If we suppose that an electric circuit in a single loop of any shape, is closed through a constant E. M. F. maintaining a steady current of c units, and that the total flux through the loop is F lines, the energy of the flux so far as it is dependent upon the current c and its local magnetic circuit, is $W = \frac{c F}{2}$

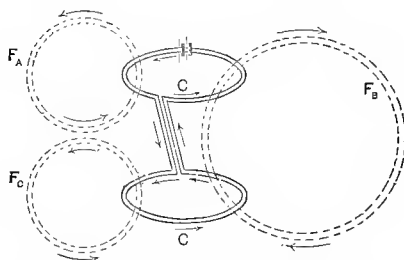


FIG. 39.—DIAGRAMMATIC ARRANGEMENT OF TWO LOOPS FORMING ONE COIL.

$$F_1 = F_A + F_B; F_2 = F_C + F_B; W = \frac{c F_1}{2} + \frac{c F_2}{2}.$$

ergs, and this energy may be practically regarded as due to the linkage of F with c , although, strictly speaking, $\frac{c F}{2}$

only yields the integral of $\frac{B^2}{8\pi}$ taken throughout the space occupied by the loop's flux. This relation of energy to current flux is diagrammatically indicated in Fig. 38.

When the electric circuit forms more than one loop round the flux, as in a coil or solenoid, Fig. 39, or when several electric circuits are placed in propinquity, such as an assemblage of helices excited by independent batteries,

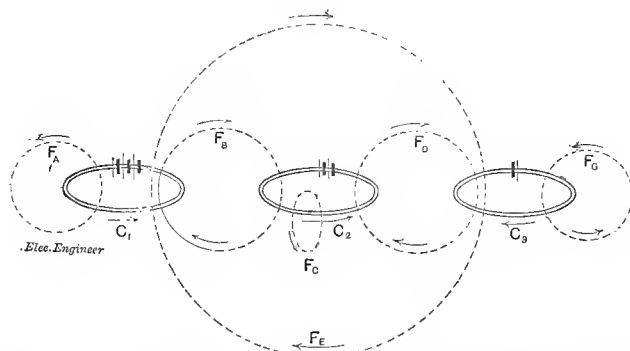


FIG. 40.—DIAGRAMMATIC ARRANGEMENT OF THREE INDEPENDENT CIRCUITS.

$$F_1 = F_a + F_b + F_c ; F_2 = F_c + F_d - F_b ; F_3 = F_b + F_d - F_a$$

$$W = \frac{c_1 F_1^2}{2} + \frac{c_2 F_2^2}{2} + \frac{c_3 F_3^2}{2}$$

Fig. 40, the same rule holds by extension. The total magnetic energy of the system is found by taking each loop in turn as though it existed alone; counting the total flux threading it in the positive direction F , and multiplying this by half the current c flowing steadily round the loop. The sum of all these products of $\frac{c F^2}{2}$ is the energy required, or

$$W = \frac{c_1 F_1^2}{2} + \frac{c_2 F_2^2}{2} + \dots + \frac{c_n F_n^2}{2} = \frac{1}{2} \sum c F^2 \text{ ergs.}$$

This equation is therefore equivalent to the statement that $\frac{\sum c F^2}{2}$ is equal to the volume integral of $\frac{B^2}{8\pi}$ extended throughout the entire space occupied by the magnetic circuit or circuits considered.

We may take a few examples by way of illustration.

If a wire of copper 100 metres long, and one mm. in radius be bent into a circular loop, and a current of three units, or 30 amperes, flows steadily round this circle, we may take it for granted that the flux urged through this loop by the M. M. F. of the current when remote from iron, or other M. M. F.'s, will amount to 600,480 lines in all. The

total magnetic energy will therefore be $\frac{3 \times 600,480}{2} =$

900,720 ergs. = 0.9 megalerg. This energy will be absorbed by the flux from the source of voltage, during the establishment of the current, will remain conserved in the medium during the steady period of current flow, and will be restored to the circuit when the current ceases.

Next consider a closed circular solenoid of N turns (1,000) carrying a current of c (1.5) units, = 15 amperes, with a mean radius of revolution $r = 20$ cms., and a small cross-sectional area of $a = 2$ sq. cms.

We know that the total flux within the solenoid will be

$$\frac{2 N c a}{r} = \frac{2 \times 1000 \times 1.5 \times 2}{20} = 300 \text{ lines, and this flux}$$

passes through each and all of the N turns, so that the summation of $\frac{c F}{2}$ is here reduced to simple multiplication and

$$W = \frac{N c F}{2} = \frac{1000 \times 1.5 \times 300}{2} = 225,000 \text{ ergs.}$$

Also the interior volume of this solenoid $V = 2 \pi r a = 251.3$ c. c. and within it, the intensity, assumed uniform, is $B = \frac{2 N c}{r} = 150$, so that the flux energy per

cubic cm. of $\frac{B^2}{8 \pi}$, amounts to $\frac{V B^2}{8 \pi} = 2 \pi r a \cdot \frac{4 N^2 c^2}{8 \pi r^2} =$

$$\frac{N^2 c^2 a}{r} = \frac{1000^2 \times 1.5 \times 1.5 \times 2}{20} = 225,000 \text{ ergs,—}$$

the same result as above, proving in this simple instance the equivalence always existing between the total voluminal energy and the product of current turns by enclosed flux $\frac{\sum c F}{2}$.

If in this case, the turns of wire forming the solenoid had been irregular, either in size or in spacing, so that some

flux had passed outside, thus causing the amount linked with each turn to be no longer uniform, it would have become necessary to make a strict summation in order to arrive at the correct amount of total energy, and

$$W = \frac{c}{2} (F_1 + F_2 + \dots + F_n).$$

Again, since the flux is always proportional to the M. M. F. and consequently to the current strength, in the absence of iron, we may consider $F_1 = c f_1$, $F_2 = c f_2$, $F_n = c f_n$, where f_1, f_2, f_n , etc., are the fluxes that would respectively thread each loop if the current strength were unity—ten amperes—, and the expression for energy then becomes

$$W = \frac{c^2}{2} (f_1 + f_2 + \dots + f_n) = \frac{c^2}{2} \Sigma f.$$

The quantity Σf which is the total number of lines linked with the circuit for unit current in the same, is called the inductance of the circuit, or sometimes the coefficient of self induction. In the closed circular solenoid just considered

$$\Sigma f = \frac{N F}{c} = \frac{2 N^2 a}{r} = \frac{2 \times 1,000 \times 1,000 \times 2}{20} = 200,000.$$

The practical unit of inductance is generally called the henry, and consists of 10^9 or one thousand million c. g. s. linkages, so that the inductance of this particular solenoid

in practical units would be $\frac{200,000}{1,000,000,000} = 2 \times 10^{-4}$ or 0.2 millihenry.

For any given coil not containing magnetizable material, the inductance Σf is a constant depending merely on its geometrical conditions, and the energy stored up in its magnetic circuit evidently varies as the square of the current through the coil.

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